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A

TREATISE

ON THE

FIRST PRINCIPLES OF ARITHMETIC,

AFTER THE METHOD OF PESTALOZZI:

WITH

Numerous Examples in all the essential Rules;

ORIGINAL AND PRACTICAL METHODS FOR CONSTRUCTING QUESTIONS;

AND

A NEW FORM FOR THE EXTRACTION OF THE CUBE ROOT.

DESIGNED FOR THE USE OF

TEACHERS AND MONITORS IN ELEMENTARY SCHOOLS.

BY

THOMAS TATE,

MATHEMATICAL MASTER OF THE NATIONAL SOCIETY'S TRAINING COLLEGE, BATTERSEA;

AUTHOR OF "EXERCISES IN ARITHMETIC," PUBLISHED UNDER THE SANCTION OF THE COMMITTEE OF COUNCIL ON EDUCATION;
A TREATISE ON "FACTORIAL ANALYSIS, WITH THE SUMMATION OF SERIES," ETC.

SECOND EDITION,

WITH ADDITIONS AND IMPROVEMENTS.

LONDON:

LONGMAN, BROWN, GREEN, AND LONGMANS,

PATERNOSTER-ROW.

1847.

TO

THE REV. THOMAS JACKSON, A.M.

PRINCIPAL

OF THE NATIONAL SOCIETY'S TRAINING COLLEGE,
BATTERSEA,

AS

A TRIBUTE OF ESTEEM FOR HIS GREAT ZEAL IN PROMOTING
INTELLECTUAL METHODS OF EDUCATION,

AND AS

A TOKEN OF GRATITUDE FOR MANY PERSONAL FAVOURS,

THIS TREATISE

ON

THE FIRST PRINCIPLES OF ARITHMETIC

IS

Dedicated,

BY

HIS MUCH OBLIGED AND MOST OBEDIENT SERVANT,

THOMAS TATE.

PREFACE.

This little work is intended to supply Teachers and Monitors of Elementary Schools, with simple and concise, yet sufficiently complete, demonstrations of the most useful Rules of Arithmetic. The rules follow each other in a logical order, no process being required in any of the questions proposed, before the rationale of such processes has been shown. Whilst the consideration of abstract principles has not been neglected, the methods of proof are strictly synthetic, and always, in the first instance, addressed to the perceptive faculties of the child. The teacher will observe, that the demonstrations of the rules should always be gone over, on the black board, before any examples upon them are given.

If a certain mechanical expertness in figures be regarded as an end, irrespective of the means by which it is attained, it must be acknowledged that the shortest and easiest way of teaching is by the dogmatic method; but if, on the contrary, we consider the study of Arithmetic from *first principles* as an important agent of mental culture, and as forming the basis of a higher education,—then the superiority of the intellectual, or demonstrative, method must be admitted by every right-minded educator.

As some acquaintance with multiplication and division is necessary to a thorough comprehension of numeration, there are certain elementary operations and properties of numbers given at the commencement of the work. A due attention to the exercises on Numeration will render many of the

subsequent demonstrations comparatively easy. The plan of teaching the compound with, what are called, the simple rules, — besides giving a simplicity, consistency, and beauty to the subject — has the powerful claim of *utility* to recommend it; for, in the present condition of society, it is too often found that the children of the poor have to leave school before they have gone through the four rules of abstract numbers, which are certainly less useful, in the business of life, than easy calculations relative to money, weights, and measures. The methods of solving questions in Rule of Three are simple, and strictly demonstrative, and therefore highly calculated to interest the mind of young persons. The Author considers that the methods which he has given for the constructing of Questions possess a decidedly practical value; for whilst the tests of accuracy are strict, and easy of application, the methods of construction are simple and expeditious. The form given for the extraction of the Cube Root is easy and practical.

While the pupil is going on with slate arithmetic, it is desirable that he should be exercised in the most useful forms of mental calculation, according to the *method* explained and illustrated in the "Exercises in Arithmetic." *

Battersea, April, 1847.

* See also "Questions in Mental Arithmetic," by Mr. McLeod.

CONTENTS.

FIRST PRINCIPLES

OF

A R I T H M E T I C.

PRELIMINARY COURSE OF ARITHMETIC.

1. Before a pupil is taught the symbols of arithmetic, it is desirable that he should understand some of the simpler operations of numbers. When he is able to count *objects* as far as twenty, such questions as the following may be given: —

1. (Writing upon the black board three marks and then two marks, ||| ||), how many are three and two?

2. How many are four and three, |||| |||?

3. (Holding up 4 fingers of one hand and 2 of the other), how many are four and two?

4. Take 2 fingers away from 5 fingers, and then, how many remain?

5. Find by counting your fingers what 5 and 3 make.

6. Here are 7 boys, and there 3 boys; how many do they make?

7. Here are 6 boys in this bench; now if I take 2 away, how many will remain?

8. Here are 5 balls, and 4 more; how many do they make?

9. Here are 10 marks; if I take 3 away, how many will be left?

10. Let James hold up his 10 fingers, and John 2 fingers; now how many do 10 and 2 make?

The teacher should extend these questions until he finds his pupils perfectly acquainted with the addition and subtraction of simple units.

Symbols and their use.

2. The sign of addition is + or plus; that of subtraction — or minus; that of multiplication × or multiply; that of division ÷ or divide; and that of equality = or equal. The following *figures* stand for the *number* of marks placed over them:—

I II III IIII IIIII IIIIII IIIIIII IIIIIIII IIIIIIIII IIIIIIIIII
0 1 2 3 4 5 6 7 8 9 10

In order to show the connection between these symbols and the operations for which they stand, the teacher should exhibit such exercises as the following on the black board:—

1. II+III=IIIII;
   2+3 = 5.
2. II+IIII=IIIIII;
   2+4 = 6.
3. III+IIII=IIIIIII;
   3+4 = 7.
4. IIIII+III=IIIIIIII;
   5+3 = 8.
5. IIIII−II=III;
   5−2 = 3.
6. IIIIII−IIII=II;
   6−4 = 2.
7. III+II+IIII=IIIIIIIII;
   3+2+4 = 9.
8. II+I+III=IIIIII;
   2+1+3 = 6.

9. I have 3 pence in one pocket, and 4 pence in the other; how many have I altogether?

Here actually putting the pence down,

● ● ● + ● ● ● ● = ● ● ● ● ● ● ●
3 + 4 = 7.

I had 5 shillings, and gave 3 of them away; how many have I left? *Ans.* 2*s.*

10. A butcher had 9 sheep, and killed 3 of them; how many had he left? *Ans.* 6.

11. Let John hold up 5 fingers, Thomas 3, and Andrew 4; now how many fingers are there altogether? *Ans.* 12.

12. A boy bought some rice for 3 pence, and an egg for 2 pence; how much should he pay altogether? *Ans.* 5*d.*

13. A person bought some oranges for 6 pence, and some pears for 3 pence; how much should he pay altogether? *Ans.* 9*d.*

14. I paid 2 pence for barley, 4 pence for meal, 3 pence for treacle; how much did I pay altogether? *Ans.* 9*d.*

15. A man had 8 pence and spent 5 pence; how much had he left? *Ans.* 3*d.*

16. A girl had 6 pence; how much should she have left after paying 4 pence for a loaf? *Ans.* 2*d.*

17. A woman bought some bacon for 9 pence; what change should the shopman give her out of a shilling? *Ans.* 3*d.*

18. John had 4 marbles. He won 5, and afterwards lost 3; how many had he then? *Ans.* 6.

19. Add 5 and 5 by counting your fingers. How many fives are there in the number of your fingers? *Ans.* 2 *fives.*

What is the half of the number of your fingers? *Ans.* 5.

If I cut a loaf into two parts and give a boy one of them, what part of the loaf will he get? *Ans. One half.*

20. How many twos have I here written? *Ans.* 3 *twos.*

|| || || = 3 twos, or 6.

How many twos are there in 6? *Ans.* 3.

What is the third of 6? *Ans.* 2.

If I cut an apple into 3 parts, and give a boy one of them, what part of the apple will he get? *Ans. one-third.*

21. How many threes have I here written. *Ans.* 4 *threes.*

||| ||| ||| ||| = 4 threes, or 12.

How many threes are there in 12? *Ans.* 4.

What is the fourth of 12? *Ans.* 3.

22. How many sixes have I here written? *Ans.* 2 *sixes.*

|||||| |||||| = 2 sixes, or 12.

How many sixes can be taken out of 12? *Ans.* 2.

What is the half of 12? *Ans.* 6.

23. Let 3 boys in the first bench hold up each 4 fingers. Now how many fours are there? *Ans. 3 fours, or* 12.

---

3. *Multiplication and Division Table.*

SECOND LINE.

| |, or 2 ones are 2;
there are 2 ones in 2; the half of 2 is 1.
|| ||, or 2 twos are 4;
there are 2 twos in 4; the half of 4 is 2.
||| |||, or 2 threes are 6;
there are 2 threes in 6; the half of 6 is 3.
|||| ||||, or 2 fours are 8;
there are 2 fours in 8; the half of 8 is 4.
And so on.

THIRD LINE.

| | |, or 3 ones are 3;
there are 3 ones in 3; the third of 3 is 1.
|| || ||, or 3 twos are 6;
there are 3 twos in 6; the third of 6 is 2.
||| ||| |||, or 3 threes are 9;
there are 3 threes in 9; the third of 9 is 3.
|||| |||| ||||, or 3 fours are 12;
there are 3 fours in 12; the third of 12 is 4.
And so on.

FOURTH LINE.

| | | |, or 4 ones, are 4;
there are 4 ones in 4; the fourth of 4 is 1.

|| || || ||, or 4 twos, are 8;
there are 4 twos in 8; the fourth of 8 is 2.

||| ||| ||| |||, or 4 threes, are 12;
there are 4 threes in 12; the fourth of 12 is 3.

|||| |||| |||| ||||, or 4 fours, are 16;
there are 4 fours in 16; the fourth of 16 is 4.

And so on.

In the same manner the remaining lines of the table may be gone through. It is desirable that the multiplication part of the table should be understood before the other part is attempted.

Examples in Multiplication and Division of units.

1. How many legs have 5 cows? *Ans. twenty.*
2. How many hands have 6 boys? *Ans. twelve.*
3. How many halfpennies are there in 4 pence? *Ans.* 8.
4. How many farthings are there in 2 pence? *Ans.* 8.
5. „ „ „ „ „ 5 pence? *Ans.* 20.
6. „ „ „ „ „ 4 pence? *Ans.* 16.
7. How many units are there in 3 tens? *Ans. thirty.*
8. „ „ „ „ 5 tens? *Ans. fifty.*
9. How many are three times 2, and 1?

|| || || + | = seven;

3 times 2 + 1 = 7

10. How many are 4 times 3, and 5? *Ans.* 17.
11. How many are 5 times 4, and 3? *Ans.* 23.
12. How many units are 2 tens, and 4? *Ans. twenty-four.*
13. „ „ „ 4 tens, and 5? *Ans. forty-five.*
14. „ „ „ 3 tens, and 2? *Ans. thirty-two.*
15. How many are 2 times 12, and 3? *Ans.* 27.

16. How many farthings are 2 pence 3 farthings? *Ans.* 11.
17. " " " 3 pence 1 farthing? *Ans.* 13.
18. How many pence could I get for 2 shillings? *Ans.* 24.
19. " " " " " 3*s.*? *Ans.* 36.
20. " " " " " 4*s.*? *Ans.* 48.
21. I bought 3 eggs for 2*d.* each; how much should I pay?

Here (actually putting down the price of each egg) two pence is the price of one egg, two pence more is the price of 2 eggs, and two pence more is the price of 3 eggs, that is 3 times 2 pence = 6 pence.

22. What should I pay the baker for 2 five-penny loaves? *Ans.* 10*d.*
23. What should you pay the milk-man for 4 pints of milk, at 2 pence a pint? *Ans.* 8*d.*
24. What should I pay for 7 oranges, at 2*d.* each? *Ans.* 14*d.*
25. Find the cost of 4 loaves, at 5*d.* each? *Ans.* 20*d.*
26. " " 3 teapots, at 6*d.* " ? *Ans.* 18*d.*
27. " " 4 books, at 7*s.* " ? *Ans.* 28*s.*
28. " " 5 chairs, at 3*s.* " ? *Ans.* 15*s.*
29. " " 4 hats, at 3*s.* " ? *Ans.* 12*s.*
30. A woman sold 3 ducks for 2 shillings each; how much did she receive? *Ans.* 6*s.*
31. A farmer sold 4 sheep for 2 sovereigns each; how much did he receive? *Ans.* 8 *sovs.*
32. How many pence in 5 sixpences? *Ans.* 30*d.*
33. " " " 6 sixpences? *Ans.* 36*d.*
34. " " " 3 fourpenny pieces? *Ans.* 12*d.*
35. " " " 5 fourpenny pieces? *Ans.* 20*d.*
36. " " " 1*s.* 3*d.*? *Ans.* 15*d.*
37. " " " 2*s.* 1*d.*? *Ans.* 25*d.*
38. " " " 2*s.* 4*d.*? *Ans.* 28*d.*
39. " " " 3*s.* 2*d.*? *Ans.* 38*d.*
40. " " " 4*s.* 3*d.*? *Ans.* 51*d.*
41. " " " 5*s.* 4*d.*? *Ans.* 64*d.*

42. How many days are there in 2 weeks? *Ans.* 14 *days.*
43. „ „ „ „ 3 weeks? *Ans.* 21 *days.*
44. „ „ „ „ 2 weeks 5 days? *Ans.* 19 *days.*
45. How many feet are there in 2 yards? *Ans.* 6 *ft.*
46. „ „ „ „ 3 yards? *Ans.* 9 *ft.*
47. „ „ „ „ 4 yards? *Ans.* 12 *ft.*
48. How many fours are there in 12?

||||, ||||, ||||, = 3 *fours.*

49. How many twos are there in 8? *Ans.* 4 *twos.*
50. How many threes are there in 15? *Ans.* 5 *threes.*
51. How many fives are contained in 20? *Ans.* 4 *fives.*
52. How many pence are there in 8 farthings? *Ans.* 2*d.*
53. „ „ „ „ 12 farthings? *Ans.* 3*d.*
54. „ „ „ „ 16 farthings? *Ans.* 4*d.*

55. How many oranges at 2*d.* each can I buy with 6*d.*? *Ans.* 3; because 2*d.* will buy 1 orange, 2*d.* more will buy 2 oranges, and 2*d.* more will buy 3 oranges; that is, 6*d.* will buy 3 oranges, or as many times as 2 can be taken out of 6 so many oranges can I buy.

56. How many slates at 3*d.* each can I buy with 12*d.*? *Ans.* 4.

57. How many kites at 5*d.* each can I buy with 15*d.*? *Ans.* 3.

58. There are 24 boys in this class; into how many rows of threes could I put you? *Ans.* 8.

59. 3 oranges cost 9*d.*; what is that for each orange? *Ans.* 3*d.*

60. I paid 8*d.* for 2 mugs; how much did I pay for each mug? *Ans.* 4*d.*

61. 4 tops cost 8*d.*; what is the price of one? *Ans.* 2*d.*
62. 6 whips cost 18d.; „ „ „ ? *Ans.* 3*d.*
63. 8 books cost 32*d.*; „ „ „ ? *Ans.* 4*d.*
64. 7 slates cost 35*d.*; „ „ „ ? *Ans.* 5*d.*
65. 5 horses cost 20*l.*; „ „ „ ? *Ans.* 4*l.*
66. 4 chairs cost 16*s.*; „ „ „ ? *Ans.* 4*s.*

67. Divide 6 marbles among 3 boys. *Ans.* *2 marbles.* What part of the 6 marbles does each boy receive? *Ans. the third.* (Why?) *Because the marbles are divided into 3 equal portions.*

68. Divide 10*d.* between 2 persons? *Ans.* 5*d. each.*

69. „ 15*d.* among 3 persons? *Ans.* 5*d. each.*

70. „ 20*d.* „ 5 persons? *Ans.* 4*d. each.*

71. What is the half of 4*d.*? *Ans.* 2*d.*

72. „ „ half of 6*d.*? *Ans.* 3*d.*

73. „ „ third of 6*d.*? *Ans.* 2*d.*

74. „ „ third of 9*d.*? *Ans.* 3*d.*

75. „ „ fourth of 8*d.*? *Ans.* 2*d.*

76. There are ten boys in this bench; what is the fifth of this number? *Ans.* 2.

77. How many tens are there in twenty? *Ans.* 2 *tens.*

78. „ „ „ in thirty? *Ans.* 3 *tens.*

79. How many shillings are there in 24*d.*? *Ans.* 2*s.*

80. „ „ „ „ 36*d.*? *Ans.* 3*s.*

81. „ „ „ „ 60*d.*? *Ans.* 5*s.*

82. How many threes can you take out of 8? *Ans.* 2 *threes and* 2 *over;* because |||, |||, || = 2 threes and 2.

83. How many fours can you take out of 11.? *Ans.* 2 *fours and* 3.

84. How many tens are there in seventeen? *Ans.* 1 *ten and* 7.

85. „ „ „ „ twenty-seven? *Ans.* 2 *tens and* 7.

86. „ „ „ „ forty-two? *Ans.* 4 *tens and* 2.

87. How many pence are there in 9 farthings? *Ans.* $2\frac{1}{4}$*d.*

88. „ „ „ „ 14 farthings? *Ans.* $3\frac{1}{2}$*d.*

89. How many shillings in 15*d.*? *Ans.* 1*s.* 3*d.*

90. „ „ „ 28*d.*? *Ans.* 2*s.* 4*d.*

91. „ „ „ 39*d.*? *Ans.* 3*s.* 3*d.*

92. How many weeks are there in 14 days? *Ans.* 2 *weeks.*

93. „ „ „ „ 21 days? *Ans.* 3 *weeks.*

94. How many yards are there in 6 feet? *Ans.* 2 *yards.*

95. „ „ „ „ 12 feet? *Ans.* 4 *yards.*

96. Put 36 counters into 3 equal groups. What operation in division does this prove?

97. Count a hundred marks in groups of tens.

Observation. The teacher should extend these questions until he finds his pupils thoroughly acquainted with the nature of the operations.

---

4. The addition, subtraction, &c. of different combinations of numbers form very important exercises.

1. Two fours and three fours make how many fours?

|||| |||| + |||| |||| |||| = five fours.

2 fours + 3 fours = 5 fours.

2. 4 fives and 3 fives make how many fives? *Ans.* 7 *fives.*

3. Let 3 boys in the first bench hold up each 10 fingers, and let 5 boys in the second bench do the same. Now how many tens do 3 tens and 5 tens make? *Ans.* 8 *tens.*

The teacher will see the importance of putting a variety of questions like this.

4. From 5 threes take away 2 threes. *Ans.* 3 *threes.*

||| ||| ||| ||| ||| — ||| ||| = 3 threes.

5 threes — 2 threes = 3 threes.

5. Let 5 boys hold up each 10 fingers. Now if we take two of these 5 tens away, how many tens will be left? *Ans.* 3 *tens; that is,* 2 *tens from* 5 *tens, and* 3 *tens remain.*

6. A man had 6 fields, and bought 3 more. How many fields had he then?

6 fields + 3 fields = 9 fields.

Precisely in the same way as we have,

6 tens + 3 tens = 9 tens.

7. A man had 7 houses, and sold 3 of them; how many had he left?

7 houses — 3 houses = 4 houses.

Precisely in the same way as we have,

7 tens — 3 tens = 4 tens.

8. How many threes are 4 times 2 threes?

• • •   • • •
• • •   • • •
• • •   • • •
• • •   • • •

Here we have 2 threes in each horizontal row; but as there are 4 of these rows, it will be readily seen that 2 threes are taken 4 times, that is, 4 times 2 threes = 8 threes. Precisely in the same way as 4 times 2 tens = 8 tens, or 4 times 2 hundreds = 8 hundreds, and so on.

9. What is the fourth of 8 threes? *Ans.* 2 *threes.*

This is shown by the preceding arrangement of counters. In precisely the same way it is shown that the fourth of 8 tens is 2 tens, and so on.

10. Let 3 boys in the first bench, 3 in the second, 3 in the third, and 3 in the fourth, each hold up 10 fingers. Now how many tens do 4 times 3 tens make? *Ans.* 12 *tens.*

11. Add 2 tens, 3 tens, and 4 tens together?

Ans. 9 *tens, or* 90.

12. Subtract 3 tens from 7 tens? *Ans.* 4 *tens, or* 40.

---

5. *Numeration of Tens and Units.*

1. I am now going to show you how to write concisely any large collection of units or ones, by the decimal notation, or notation of tens.

The *number* of marks, |||||||||| + |||, are written,
1 ten + 3 units, or 13.

But 1 ten and 3 make thirteen, therefore 13 may be read in two ways; first it is 1 ten and 3 units, second it is thirteen.

2. The *number* of marks, |||||||||| |||||||||| + ||||,
are written, 2 tens, + 4 units, or 24.

But 2 tens and 4 units make twenty-four; therefore 24 may be read in two ways; first it is 2 tens and 4 ones, second it is twenty-four.

3. Let 4 boys in the first bench each hold up his 10 fingers, and let 1 boy in the second bench hold up 5 fingers. Now what shall I write down for the number of fingers? *Ans. Put* 5 *down in the one's place, and* 4 *in the ten's place; that is,* 45.

You say that this reads 4 tens and 5 units; but it may be read in another way. *Ans. It will also read forty-five.* (Why?) *Because* 4 *tens make forty, and* 5 *ones more make forty-five.*

In this manner the teacher may proceed until his pupils are perfectly acquainted with the notation of tens and units.

---

6. *Numeration of Hundreds, Tens, and Units.*

In order to give the pupils an adequate idea of hundreds, let the teacher place 10 boys in each bench; then he may proceed with the following exercises:—

1. Let the boys in the first bench hold up all their fingers. How many tens have we? *Ans.* 10 *tens, or* 1 *hundred.* Now as there are no units, I put 0 in the units' place and 10 in the tens' place, thus 100; this then will stand for 10 tens, or 1 hundred. Here the 1, in order to read hundreds, is put in the third place.

2. Let the boys in the two first benches hold up all their fingers. Now how many hundreds are there? (*Ans.* 2 *hundreds.*) I therefore write 2 in the third place of figures, that is, 200 stands for 2 hundreds. And so on to any number of hundreds.

3. Let the boys in the 3 first benches hold up all their fingers, 2 boys in the fourth bench all their fingers, and 1 boy in the fifth bench 5 of his fingers. Now how many fingers have we? *Ans.* 3 *hundreds,* 2 *tens, and* 5 *units.* To write this number, therefore, I put 5 in the units' place, 2 in the tens' place, and 3 in the hundreds' place, that is, 325.

But we may also read this in tens. *Ans. It will also read*

32 *tens, and* 5 *units.* (Why ?) *Because the* 3 *hundreds make* 30 *tens, and the* 2 *tens more make* 32 *tens.*

Proceeding in this way the teacher may exhibit every variety of form that can arise out of the different combinations of hundreds, tens, and units; viz., 438, 570, 300, 207.*

Having explained the decimal notation as far as hundreds, the notation of thousands, &c., will follow as an easy induction.

The following exercise exhibits one of the most useful properties of the decimal notation.

To resolve Tens into Units, and conversely.

Commencing, for example, with 6 tens, we have

6 tens or 60 = sixty
6 tens + 1 or 61 = sixty-one
6 tens + 2 or 62 = sixty-two
: : :
7 tens or 70 = seventy
7 tens + 1 or 71 = seventy-one
7 tens + 2 or 72 = seventy-two,
and so on.

To resolve Hundreds into Tens, and conversely.

100 = 10 tens
110 = 10 tens + 1 ten = 11 tens
120 = 10 tens + 2 tens = 12 tens
: : :
200 = 2 times 10 tens = 20 tens
210 = 2 times 10 tens + 1 ten = 21 tens
: : :
300 = 3 times 10 tens = 30 tens
310 = 3 times 10 tens + 1 ten = 31 tens
320 = 3 times 10 tens + 2 tens = 32 tens.
and so on.

* The Bishop of St. Asaph calls these 4 forms, taken in connection with the following, 47, 20, and 5, the seven varieties of notation.

In precisely the same way thousands are converted into hundreds, and so on.

7. The teacher will now have shown how the ten characters 0, 1, 2, 3, . . . 9, may be made, by relative position, to express any number whatever; that figures in the first right hand place stand for units, in the second place for tens, in the third place for hundreds, in the fourth place for thousands, and so on; and that any figure in the tens' place is 10 times the value of the same figure in the units' place, any figure in the hundreds' place is 10 times the value of the same figure in the tens' place, and so on. It will also be understood that although the cipher 0 has no value in itself, yet it serves to affect the relative value of figures. Thus in order to express 4 hundreds, all that we have to do is to put 4 in the hundreds' *place;* but as we have neither tens, nor units, we necessarily affix two noughts. In like manner we have 6 thousands and fifty expressed by 6050, where the first cipher changes the 5 to tens, and the second cipher, removing the figure 6 a place to the left, gives it the value of thousands instead of hundreds, which it would otherwise have.

EASY QUESTIONS IN THE FOUR ELEMENTARY OPERATIONS.

8. The teacher should be able to give simple exercises, like the following, without the aid of a book containing questions with their answers.

Addition.

1. Add together 27 marbles and 25 marbles.

tens.	units.		tens.		units.
2	7	or	....	+	....
2	5		....	+	......
5	2		5 tens	+	2

Here the units put together make 12, which are equal to

1 ten and 2 units; we therefore put 2 down in the column of units, and carry or reserve the 1 ten to be put to the tens; then 2 tens and 2 tens make 4 tens, and the 1 ten, which we got out of the units, makes 5 tens.

2. How many do 34 oranges and 23 oranges make? *Ans.* 57.

3. How many do 46 nuts and 38 nuts make? *Ans.* 84.

4. Add together 6*s.* 4*d.* and 2*s.* 5*d.*

s.	*d.*
6	4
2	5
8	9

Here the pence put together make 9*d.*, and the shillings put together make 8*s.*

5. Add together 5*s.* 2*d.* and 3*s.* 6*d.* *Ans.* 8*s.* 8*d.*

6. Add together 4*s.* 9*d.* and 2*s.* 7*d.*

s.	*d.*
4	9
2	7
7	4

Here 7*d.* and 9*d.* make 16*d.* Now we have to take the shillings out of 16*d.*, which will be 1*s.* and 4*d.*, we therefore put down 4*d.* in the column of pence, and reserve the 1*s.* to be put to the shillings; then 2*s.* and 4*s.* make 6*s.*, and the 1*s.* which we got out of the pence make 7*s.*

7. Add together 6*s.* 7*d.*, and 2*s.* 6*d.* *Ans.* 9*s.* 1*d.*

8. A person bought a hat for 6*s.* 8*d.*, and a cap for 2*s.* 6*d.*; how much will he have to pay altogether? *Ans.* 9*s.* 2*d.*

9. A woman sold a chicken for 2*s.* 3*d.*, and a goose for 7*s.* 10*d.*; how much should she receive altogether? *Ans.* 10*s.* 1*d.*

Subtraction.

1. Subtract or take 18 marbles from 34 marbles.

tens.	units.		tens.	units.
3	4	or	....	+ ....
1	8		....	+ ....
1	6		1 ten.	+ 6

Here, as we cannot take 8 units from 4 units, we take or "borrow" one of the tens from the 3 tens, which, put to the 4 make 14, then 8 from 14 and 6 remain; we have now 1 ten to take from 2 tens, which leaves 1 ten.

2. Take 21 boys from 39 boys. *Ans.* 18 *remain.*

3. Take 26 nuts from 54 nuts. *Ans.* 28 *remain.*

4. Take 3*s.* 2*d.* from 8*s.* 5*d.*

s.	*d.*
8	5
3	2
5	3

Here the 2*d.* taken from the 5*d.* leaves 3*d.*, and the 3*s.* from the 8*s.* leaves 5*s.*

5. Take 4*s.* 5*d.* from 7*s.* 9*d.* *Ans.* 3*s.* 4*d.*

6. Take 3*s.* 8*d.* from 6*s.* 2*d.*

s.	*d.*
6	2
3	8
2	6

Here, as we cannot subtract 8*d.* from 2*d.*, we must take or "borrow" one of the shillings from the 6*s.*, and put it to the 2*d.*, which will make 14*d.*, and then 8*d.* from 14*d.* leaves 6*d.*; we have now 3*s.* to take from 5*s.* (Why not 6*s.*?) leaving 2*s.*

7. Take 9*d.* from 1*s.* 4*d.* *Ans.* 7*d.*

8. John has 1*s.* 7*d.* and James has 10*d.*; how much has John more than James? *Ans.* 9*d.*

9. A person had 5*s.* 3*d.*, and paid 2*s.* 8*d.* for sugar; how much had he left? *Ans.* 2*s.* 7*d.*

Multiplication.

1. Take 34 two times.

tens.	units.
3	4
	2
6	8

Here we have 4 units to take 2 times, and also 3 tens to take 2 times. Then 2 times 4 units are 8 units, and 2 times 3 tens are 6 tens.

2. Take 43 two times. *Ans.* 86.

3. Multiply 24 by 3.

tens.	units.	
2	4	Here 3 times 4 units are 12 units, which
	3	equal 1 ten and 2 units; we therefore set down
7	2	2, and carry or reserve 1 ten; then 3 times 2

tens are 6 tens, and the 1 ten which we got out of the units, make 7 tens.

To illustrate this operation, let 2 boys in the first bench hold up all their fingers, and let 1 boy in the same bench hold up 4 fingers. Let the same thing be done with the boys in the second and third benches. Then we shall have 3 times 24 fingers exhibited to the class.

4. Multiply 37 by 2. *Ans.* 74.

5. Three boys have 14 marbles each; how many have they altogether? *Ans.* 42.

6. Take 2*s.* 4*d.* two times.

s.	*d.*	
2	4	Here the 4*d.* taken 2 times give 8*d.*, and the
	2	2*s.* taken 2 times give 4*s.*
4	8	

7. Take 3*s.* 2*d.* three times. *Ans.* 9*s.* 6*d.*

8. What will be the cost of 2 hats at 4*s.* 3*d.* each? *Ans.* 8*s.* 6*d.*

9. Take 2*s.* 5*d.* three times.

s.	*d.*	
2	5	Here 5*d.* taken 3 times give 15*d.*, which are
	3	1*s.* and 3*d.*, we therefore set down the 3*d.*, and
7	3	carry or reserve the 1*s.*; then 2*s.* taken 3 times

give 6*s.*, and the 1*s.*, which we got out of the pence, make 7*s.*

10. Take 2*s.* 5*d.* four times. *Ans.* 9*s.* 8*d.*

11. What will be the cost of 2 books at 3*s.* 8*d.* each? *Ans.* 7*s.* 4*d.*

12. What will be the cost of 3 baskets at 2*s.* 6*d.* each? *Ans.* 7*s.* 6*d.*

13. In £3 how many shillings? *Ans.* 60*s.*

14. In £4 how many shillings? *Ans.* 80*s.*

15. In £2 9*s.* how many shillings? *Ans.* 49*s.*

16. Find the amount of the following bill:

		s.	d.		s.	d.
2 penknives	at	1	6	each		
3 table spoons	at	2	4	each		
4 inkstands	at	1	8	each		
					16	8

Division.

1. What is the half of 46?

	tens.	units.
2)	4	6
	2	3

Here the half of 4 tens is 2 tens, and the half of 6 units is 3 units.

2. What is the half of 86? *Ans.* 43.

3. Divide 63 nuts among 3 boys, how many will each boy receive? *Ans.* 21.

What part of the 63 nuts will each receive?

Ans. The third.

4. What is the fourth of 96?

	tens.	units.
4)	9	6
	2	4

Here the fourth of 9 tens is 2 tens with 1 ten over, this ten put to the 6 units will make 16 units, then the fourth of 16 units is 4 units.

5. What is the third of 72? *Ans.* 24.

6. What is the half of 4*s.* 8*d.*?

	s.	d.
2)	4	8
	2	4

Here the half of 4*s.* is 2*s.*, and the half of 8*d.* is 4*d.*

7. What is the third of 6*s.* 9*d.* *Ans.* 2*s.* 3*d.*

8. Divide 8*s.* 4*d.* between 2 persons. *Ans.* 4*s.* 2*d. each.*

9. What is the third of 7*s.* 6*d.*?

	s.	d.
3)	7	6
	2	6

Here the third of 7*s.* is 2*s.* with 1*s.* over, this 1*s.* put into pence gives 12*d.*, which, added to the 6*d.*, give 18*d.*, then the third of 18*d.* is 6*d.*

10. What is the fourth of 9*s*. 8*d*.? *Ans*. 2*s*. 5*d*.

11. 3 slates cost 4*s*. 6*d*.; what is the cost of 1? *Ans*. 1*s*. 6*d*.

12. 4 mugs cost 5*s*. 4d.; what is the cost of 1? *Ans*. 1*s*. 4*d*.

13. In 28*s*. how many pounds? *Ans*. £1 8*s*.

14. In 47*s*. how many pounds? *Ans*. £2 7*s*.

15. Divide 8*s*. 3*d*. among 3 persons; how much should each person have? *Ans*. 2*s*. 9*d*.

Mixed Questions.

1. A farmer sold 3 geese at 6*s*. 6*d*. each, and bought 2 mugs at 1*s*. 8*d*. each. What money did he take home? *Ans*. 16*s*. 2*d*.

2. A man took 16*s*. to market. He bought 2 knives at 8*d*. each, and 6 plates at 1*s*. 3*d*. each. How much money had he remaining? *Ans*. 7*s*. 2*d*.

3. A woman sold 9 lbs. of butter for 1*s*. 2*d*. a lb., and with the money purchased 2 lbs. of tea at 4*s*. 8*d*. a lb. How much money had she left? *Ans*. 1*s*. 2*d*.

4. A man bought 4 chairs at 3*s*. 4*d*. each, and 2 tables at 8*s*. 9*d*. each. How much more did he pay for the tables than the chairs? *Ans*. 4*s*. 2*d*.

5. 3 lbs. of coffee cost 7*s*., and 4 lbs. of tea cost 13*s*. 4*d*. How much more did a lb. of tea cost than a lb. of coffee? *Ans*. 1*s*.

6. A person had 9*s*.; after paying for 2 lbs. of tea at 3*s*. a lb., he found that the remainder of his money would just purchase 4 lbs. of sugar. How much was the sugar a lb.? *Ans*. 9*d*.

ADDITION.

9. Add together 237, 428, and 569.

hund.	tens.	units.
2	3	7
4	2	8
5	6	9
12	3	4

Here the addition of the units produces 24, which are equal to 2 tens and 4 units; we therefore put down the 4 in the units' place, and carry the 2 tens to the column of tens, the addition of which produces 13 tens or 1 hundred and 3 tens, we therefore put down the 3 in the tens' place, and carry the 1 hundred to the column of hundreds, and so on.

Such questions as the following may be put in the course of the demonstration:—

Teacher. Why do you put the units into tens?

Pupil. To add the tens so found to the column of tens.

Teacher. How many of the tens in the second column go to form 1, that is a hundred, in the third column?

Pupil. Ten; because 10 tens make 1 hundred.

10. The same principle applies to the addition of pounds, shillings, and pence, as that which we have observed in the addition of units, tens, and hundreds.

	£	*s.*	*d.*
Ex.	3	8	9
	6	7	4
	5	6	8
	2	5	9
	£17	8	6

Here the addition of the column of pence gives 30*d.*, which are 2*s.* and 6*d.*, we therefore put down the 6 in the column of pence, and carry the 2*s.* to the column of shillings, the addition of which produces 28*s.*, which are £1 and 8*s.*; we therefore put down the 8 in the column of shillings, and carry the £1. to the column of pounds, the addition of which produces £17.

Teacher. How do you put the pence into shillings?

Pupil. By counting 1*s.* for every 12*d.*, that is, as many twelves as I can take out of my pence, so many shillings must I have.

Teacher. In what respect do the columns here differ from those in the last example?

It will be found that the addition of simple sums of money is more easily understood by children than the addition of abstract numbers. The fact is, a child is more familiar with pence, shillings, and pounds, than it is with the abstractions, units, tens, and hundreds. A teacher may give a great interest to such questions, by actually putting down the pence and shillings, which he is about to add, telling his pupils, at the same time, that as a Grocer has separate apartments in his cash-box, for his pence, shillings, and sovereigns, so we have separate columns for our pence, shillings, and pounds.

Examples in Addition.

1. Edward had 23 marbles in one bag, and 182 in another; how many had he altogether? *Ans.* 205.

2. A farmer had 3 flocks of sheep. The first contained 146, the second 263, and the third 35. How many sheep had he? *Ans.* 444.

3. There were 4 chests of oranges. The first contained 527, the second 265, the third 69, and the fourth 72; how many oranges were there altogether? *Ans.* 933.

4. A person paid 6*s.* 4*d.* for coffee, 8*s.* 6*d.* for tea, and 7*d.* for rice; what did he pay altogether? *Ans.* 15*s.* 5*d.*

5. A butcher bought 3 oxen, for the first he paid £18 10*s.*, for the second £13 17*s.*, and for the third £15. 13*s.*; what did he pay for the whole? *Ans.* £48.

6. A merchant bought 3 lots of goods, for the first he paid £6 9*s.* 8*d.*, for the second £1. 8*s.* 5*d.*, and for the third £1 7*s.* 2*d.*; what had he to pay for the whole? *Ans.* £9. 5*s.* 3*d.*

7. Add together three hundred and five, two hundred and sixty-four, and forty-nine. *Ans.* 618.

8. Add together 57, 245, and 34. *Ans.* 336.

9. Add together 46, 350, 32, and 7. *Ans.* 435.

10. Add together 106, 23, 6, and 95. *Ans.* 230.

11. Add together £7 5*s.*, £1 19*s.*, and 6*s.* 7*d.*? *Ans.* £9. 10*s.* 7*d.*

(1.)	(2.)	(3.)	(4.)
2574	3861	143856	256743
396	99	25974	105894
1089	1584	341658	3996

(5.)	(6.)	(7.)	(8.)
4653	1287	238761	520842
1485	396	147852	684838
693	1089	358641	757654
2475	99	89910	878798

Answers.

(1.)	4059	(2.)	5544	(3.)	511488	(4.)	366633
(5.)	9306	(6.)	2871	(7.)	835164	(8.)	2842132

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
(1.)	5	3	7	(2.)	4	13	5	(3.)	14	9	8¼	(4.)	24	5	11
	2	9	8		2	17	7		6	8	7½		64	8	10
	1	7	5		1	19	9		19	7	6¾		78	9	8

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
(5.)	7	9	8	(6.)	8	19	2½	(7.)	15	5	9½	(8.)	26	8	10
	5	8	7		5	16	9¾		8	9	7¾		33	5	11
	8	6	9		0	17	5½		19	7	0½		14	7	10
	9	7	6		7	15	8¾		13	6	2¼		98	9	11

Answers.

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
(1.)	9	0	8	(2.)	9	10	9	(3.)	40	5	10½	(4.)	167	4	5
(5.)	30	12	6	(6.)	23	9	2½	(7.)	56	8	8	(8.)	172	12	6

SUBTRACTION.

11. In order that the difference of two numbers may remain the same, we must increase the two numbers by the same quantity, for example,

$$||||| - ||| = ||, \text{ or } 5 - 3, = 2;$$

and increasing each of the numbers by 1,

$$|||||| - |||| = ||, \text{ or } 6 - 4, = 2.$$

In like manner,

5 tens — 3 tens = 2 tens,

and increasing each of the numbers by 1 ten,

6 tens — 4 tens = 2 tens.

This axiom will enable us to explain the rule of Subtraction.

12. Subtract 356 from 634.

hund.	tens.	units.
6	3	4
3	5	6
2	7	8

Here, as we cannot take 6 from 4, we take or "borrow" one of the tens from the 3 tens, and then 6 from 14 and 8 remain; we have now 5 tens to take from 2 tens; (why not from 3 tens?) as this cannot be done, we take or borrow a hundred or 10 tens, from the 6 hundreds, and then we have 5 tens from 12 tens and 7 tens remain; lastly, we have 3 hundreds from 5 hundreds and 2 hundreds remain.

In the course of the foregoing demonstration we had 5 tens to subtract from 12 tens, now the result will not be altered if we increase each of the numbers by 1 ten, that is, if we say 6 tens from 13 tens; in like manner in the place of saying 3 hundreds from 5 hundreds we may say, without altering the result, 4 hundreds from 6 hundreds: thus establishing the common rule of subtraction, that when we "borrow" 1 from a figure in the top line we must "carry" 1 to the next figure in the bottom line.

Teacher. Why do we borrow 1 ten from the 3 tens?

Pupil. To enable us to subtract the units.

Teacher. Why do we say 6 tens from 13 tens?

Pupil. Because this gives the same remainder as 5 tens from 12 tens.

Teacher. Why then is the form of "borrow" and "carry" used?

Pupil. Because it is thought more convenient in practice.

13. Subtract £4 12*s.* 9*d.* from £9 5*s.* 2*d.*

	£	*s.*	*d.*
Ex.	9	5	2
	4	12	9
	4	12	5

As we cannot take 9*d.* from 2*d.* we must take, or "borrow," one of the shillings from the 5*s.* and then 9*d.* from 14*d.* and 5*d.* remain; we have now 12*s.* to take from 4*s.*; as this cannot be done, we take or borrow £1 or 20*s.* from the £9, and then we have 12*s.* from 24*s.*, or what is the same thing, 13*s.* from 25*s.*, and 12*s.* remain; lastly, we have £4 to take from £8, or what is the same thing, £5 from £9 and £4 remain.

Examples in Subtraction.

1. A man took 135 eggs to market, and sold 87 of them. How many did he carry home? *Ans.* 48.

2. A farmer had 420 sheep, and sold 134 of them. How many had he left? *Ans.* 286.

3. A boy had 103 marbles, and lost 86 of them. How many had he left? *Ans.* 17.

4. A man was born in 1788, what age will he be in 1849? *Ans.* 61.

5. A chest of oranges contained 703; but 285 were bad. How many good ones were there? *Ans.* 418.

6. A man had £1 3*s.* 6*d.* in his pocket, and paid a bill of 9*s.* 8*d.* How much had he left? *Ans.* 13*s.* 10*d.*

7. A person had £63 5*s.* a year, and spent £48 9*s.* How much did he save? *Ans.* £14 16*s.*

8. A person had £46 4*s.*, but he owed £9 8*s.* How much money was he worth? *Ans.* £36 16*s.*

9. From three hundred and two take one hundred and twenty-five. *Ans.* 177.

(1.) 1485	(2.) 2574	(3.) 647352	(4.) 821067
297	1683	253746	134754
(5.) 6204	(6.) 4257	(7.) 824175	(8.) 523143
3036	198	131868	134865

Answers.

(1.) 1188	(2.) 891	(3.) 393606	(4.) 686313
(5.) 3168	(6.) 4059	(7.) 692307	(8.) 388278

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	5	6	5	(2.)	24	3	9	(3.)	36	16	$2\frac{1}{2}$
	1	2	8		8	14	2		18	0	$6\frac{1}{4}$
(4.)	6	3	2	(5.)	8	3	4	(6.)	34	13	0
	2	9	8		2	9	$8\frac{1}{2}$		19	16	8
(7.)	45	12	$3\frac{1}{4}$	(8.)	2	7	0	(9.)	215	2	3
	38	18	$2\frac{1}{2}$		1	8	9		154	14	8
(10.)	306	5	2	(11.)	63	10	$5\frac{1}{2}$	(12.)	3	4	5
	74	2	4		54	17	$2\frac{3}{4}$		2	3	8

Answers.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	4	3	9	(2.)	15	9	7	(3.)	18	15	$8\frac{1}{4}$
(4.)	3	13	6	(5.)	5	13	$7\frac{1}{2}$	(6.)	14	16	4
(7.)	6	14	$0\frac{3}{4}$	(8.)	0	18	3	(9.)	60	7	7
(10.)	232	2	10	(11.)	8	13	$2\frac{3}{4}$	(12.)	1	0	9

MULTIPLICATION.

14. When a number is to be repeated, or added to itself, a certain number of times, the process is called Multiplication, as for example, $4 + 4 + 4 = 3$ times 4, or 4×3.

15. The process of Multiplication is founded upon the axiom, that any quantity taken a certain number of times is the same as the parts of that quantity taken the same number of times; for example,

$$3 \text{ times } 27 = 3 \text{ times } 20 + 3 \text{ times } 7.$$

The truth of this axiom will be rendered apparent by arranging counters, as in the following figure:

● ● ●   ● ●
● ● ●   ● ●
● ● ●   ● ● $= 4$ times $5 = 4$ times $3 + 4$ times 2.
● ● ●   ● ●

In the same manner the teacher may illustrate any other case.

16. Multiply 245 by 3.

hund.	tens.	units.
2	4	5
		3
7	3	5

In this example, the units taken 3 times give 15, which equal 1 ten and 5 units, we therefore set down 5, and carry or reserve 1 ten; the tens taken 3 times give 12 tens, and 1 ten which we reserved give 13 tens; we therefore set down 3 tens, and reserve 10 tens or 1 hundred; the hundreds taken 3 times give 6 hundreds, and the 1 hundred which we reserved give 7 hundreds.

17. Multiply £2 8*s.* 5*d.* by 4.

£	*s.*	*d.*
2	8	5
		4
9	13	8

Here the pence taken 4 times give 20*d.*, which are 1*s.* and 8*d.*, we therefore set down the 8*d.* and carry or reserve the 1*s.*; the shillings taken 4 times give 32*s.*, and 1*s.* which we reserved give 33*s.*, which are £1 and 13*s.*; we

therefore set down the 13*s.* and reserve the £1; the pounds now taken 4 times give £8, which added to the £1 reserved, give £9.

Examples in Multiplication with One Figure in the Multiplier.

1. How many nuts are there in 3 bags, when each bag contains 232? *Ans.* 696.

2. How many oranges are there in 4 chests, when each chest contains 643? *Ans.* 2572.

3. If a man earn £257 a year, how much will he earn in 5 years? *Ans.* £1285.

4. How many farthings are there in 376*d.*? *Ans.* 1504.

5. In a drove of 495 cattle how many feet are there? *Ans.* 1980.

6. A parish contains 1572 houses; now if each house holds 6 people, how many people are there in the parish? *Ans.* 9432.

7. A gentleman gave 7*d.* each to 235 poor persons; how many pence did he give away? *Ans.* 1645.

8. How many days are there in 306 weeks? *Ans.* 2142.

9. How many pence are there in 276*s.*? *Ans.* 3312.

10. A draper bought 250 pieces of calico; each piece measured 9 yards; how many yards were there altogether? *Ans.* 2250.

11. In 1 fathom there are 6 feet; how many feet are there in 378 fathoms? *Ans.* 2268.

12. A bricklayer worked 9 hours a day for 313 days; how many hours did he work? *Ans.* 2817.

13. What will be the cost of 5 tons of coals at £1 12*s.* 4*d.* for every ton? *Ans.* £8 1*s.* 8*d.*

14. 7 cwts. of cheese at £2 6*s.* per cwt.? *Ans.* £16 2*s.*

15. 8 cwts. of sugar at £2 9*s.* 3*d.* per cwt.? *Ans.* £19 14*s.*

16. 9 horses at £5 10*s.* each? *Ans.* £49 10*s.*

17. 6 chairs at £1 7*s.* each? *Ans.* £8 2*s.*

18. 5 cows at £3 19*s.* each? *Ans.* £19 15*s.*

19. 11 sheep at £1. 14s. 6d. each? *Ans.* £18. 19s. 6d.
20. 4 tables at £6. 5s. 9d. each? *Ans.* £25. 3s.
21. 10 books at £2. 3s. 8d. each? *Ans.* £21. 16s. 8d.

(1.)	1245 × 2	(2.)	2574 × 3	(3.)	6042 × 4
(4.)	23578 × 5	(5.)	106893 × 6	(6.)	20879 × 7
(7.)	30075 × 8	(8.)	435627 × 9	(9.)	853 × 10
(10.)	25839 × 11	(11.)	632406 × 12	(12.)	12809 × 12.

Answers.

(1.)	2490	(2.)	7722	(3.)	24168	(4.)	117890
(5.)	641358	(6.)	146153	(7.)	240600	(8.)	3920643
(9.)	8530	(10.)	284229	(11.)	7588872	(12.)	153708

	£	s.	d.			£	s.	d.	
(1.)	7	5	2	× 2	(2.)	15	6	4	× 3
(3.)	16	8	9	× 4	(4.)	17	10	8	× 4
(5.)	18	7	$8\frac{1}{4}$	× 5	(6.)	20	18	6	× 6
(7.)	28	8	$4\frac{1}{2}$	× 7	(8.)	27	17	$5\frac{1}{2}$	× 8
(9.)	124	18	10	× 9	(10.)	54	7	$11\frac{3}{4}$	× 10
(11.)	37	5	$9\frac{1}{2}$	× 11	(12.)	34	0	$5\frac{1}{2}$	× 12

Answers.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	14	10	4	(2.)	45	19	0	(3.)	65	15	0
(4.)	70	2	8	(5.)	91	18	$5\frac{1}{4}$	(6.)	125	11	0
(7.)	198	18	$7\frac{1}{2}$	(8.)	222	19	8	(9.)	1124	9	6
(10.)	543	19	$9\frac{1}{2}$	(11.)	410	3	$8\frac{1}{2}$	(12.)	408	5	6

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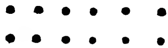
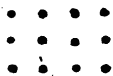
17. *To resolve numbers into factors.* Children may be aided in finding the factors of any number by the following

method. If we want, for example, to find the different factors of 12, we take 12 counters and arrange them so as to form as many rectangles as possible; thus we have,

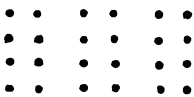
I.  this is $6 \times 2 = 2 \times 6 = 12$.

II.  this is $4 \times 3 = 3 \times 4 = 12$.

The teacher will find this exercise highly amusing and instructive to children, more especially if they be allowed to discover the different arrangement for themselves.

1. What are the factors of 18? *Ans.* 2 *and* 9, 3 *and* 6.
2. ,, ,, ,, 24? *Ans.* 4 *and* 6, 3 *and* 8, 2 *and* 12.
3. ,, ,, ,, 36? *Ans.* 4 *and* 9, 6 *and* 6, 3 *and* 12.
4. ,, ,, ,, 21? *Ans.* 3 *and* 7.

18. We may multiply one number by another, by successively multiplying by the factors of the multiplier: thus, since 6 are 3 times 2, we may find the product of 4 by 6, by first multiplying 4 by 2, and then multiplying this product by 3.

Proof.  this is 4×6 or $4 \times 2 \times 3$.

As a further illustration of this important principle, let it be required to find the cost of 18 yards of cloth at £2 7*s.* 2*d.* each yard.

£	*s.*	*d.*	
2	7	2	cost of 1 yard
		3	
7	1	6	cost of 3 yards
		6	
42	9	0	cost of 6 times 3 yards, or 18 yards.

Here we have to take £2 7*s.* 2*d.* 18 times; but as it is inconvenient to multiply at once by 18, we successively multiply by its factors 3 and 6, or 2 and 9. Multiplying the cost of 1 yard by 3 gives us the cost of 3 yards; then, as the cost of 18 yards must be 6 times as much as the cost of 3 yards, we take the cost of 3 yards 6 times to find the cost of 18 yards.

19. When we cannot exactly find the factors of the multiplier, we proceed as in the following example:

Required the cost of 31 articles at £2 8*s.* 4*d.* each.

£	*s.*	*d.*	
2	8	4	
		3	
7	5	0	cost of 3 articles
		10	
72	10	0	cost of 30 articles
2	8	4	cost of 1 article
74	18	4	cost of 31 articles.

Here adding the cost of 1 article to the cost of 30, must give us the cost of 31.

20. To multiply by 10, 100, 1000, &c.

Ex.
```
 345
  10
----
3450
```
Here multiplying by 10 will make the units tens, the tens hundreds, the hundreds thousands, and so on. It therefore appears that we multiply by 10 by simply affixing a cypher to the multiplicand. Again, since 100 are 10 times 10, we multiply by 100 by simply affixing two cyphers to the multiplicand, and so on.

21. To multiply by tens, hundreds, &c.

Ex. 1.
```
  344
   60
-----
20640
```
As 60 are 10 times 6, we here first multiply by 6 and then by 10, which latter operation is effected by simply adding a cypher.

Ex. 2.
```
  423
  500
──────
211500
```
Here as 500 are 100 times 5, we first multiply by 5, and then by 100.

Ex. 3.

3462	
723	
10386	multiplicand taken 3 times
69240	„ „ 20 times
2423400	„ „ 700 times
2503026	total product

As 723 are made up of 3 + 20 + 700, we first take the multiplicand 3 times, then 20 times, and lastly 700 times, and then add the products together for the total product. It is obvious that the cyphers in the first and second columns may be left out without vitiating the result.

Examples in Multiplication.

1. How many oranges are there in 24 chests, when each chest contains 562? *Ans.* 13488.

2. If a man walk 36 miles in 1 day; how many miles will he walk in 49 days? *Ans.* 1764.

3. What will be the cost of 14 maps at 14*s.* 8*d.* each? *Ans.* £10 5*s.* 4*d.*

4. 48 lbs. of coffee at 2*s.* 2*d.* a lb. *Ans.* £5 4*s.*

5. 16 oxen at £3 17*s.* 4*d.* each. *Ans.* £61 17*s.* 4*d.*

6. 32 yards of cloth at 4*s.* 9*d.* a yard. *Ans.* £7 12*s.*

7. 42 lambs at 11*s.* 6*d.* each. *Ans.* £24 3*s.*

8. 28 yards of silk at 3*s.* 7*d.* a yard. *Ans.* £5 0*s.* 4*d.*

9. 15 yards „ „ 2*s.* 3*d.* „ *Ans.* £1 13*s.* 9*d.*

10. 16 hats at 4*s.* 1½*d.* each. *Ans.* £3 6*s.*

11. If the cost of 1 article is £2 7*s.* 4*d.*, what is the cost of 21? *Ans.* £49 14*s.*

12. 25 articles at £5 9*s.* 8¼*d.* each. *Ans.* £137 2*s.* 2¼*d.*

13. 54 at £3 7*s.* 6½*d.* each. *Ans.* £182 7*s.* 3*d.*

14. 96 at £2 5*s.* 2*d.* each. *Ans.* £216 16*s.*

15. If a man earn 4*s.* 8*d.* a day, how much will he earn in 27 days? *Ans.* £6 6*s.* 0*d.*

16. What is the value of a firkin of butter containing 56 lbs., at 10*d.* per lb.? *Ans.* £2 6*s.* 8*d.*

17. If 1 yard of velvet cost £1 13*s.* 4*d.*, how much will 35 yards cost? *Ans.* £58 6*s.* 8*d.*

18. A railway train runs 37 miles an hour; what distance will it run in 48 hours? *Ans.* 1776 miles.

(1.) 1648724 × 18	(2.) 3579642 × 24	(3.) 2479752 × 15
(4.) 6059394 × 16	(5.) 1357642 × 36	(6.) 3289671 × 21
(7.) 1036563 × 33	(8.) 2478642 × 27	(9.) 1256541 × 63

Answers.

(1.) 29677032	(2.) 85911408	(3.) 37196280
(4.) 96950304	(5.) 48875112	(6.) 69083091
(7.) 34206579	(8.) 66923334	(9.) 79162083.

	£	s.	d.				£	s.	d.		
(1.)	3	7	4	×	16	(2.)	22	4	5	×	15
(3.)	25	9	6	×	24	(4.)	36	12	2	×	22
(5.)	17	15	7	×	35	(6.)	29	17	4	×	32
(7.)	5	4	2¼	×	18	(8.)	20	5	6¾	×	48
(9.)	42	18	10½	×	49	(10.)	45	18	5¼	×	60
(11.)	56	5	8¾	×	96	(12.)	23	14	10	×	56

Answers.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	53	17	4	(2.)	333	6	3	(3.)	611	8	0
(4.)	805	7	8	(5.)	622	5	5	(6.)	955	14	8
(7.)	93	15	4½	(8.)	973	7	0	(9.)	2104	4	10½
(10.)	2755	6	3	(11.)	5403	10	0	(12.)	1329	10	8

---

1. 31 cows at £4 7*s.* 4*d.* each. *Ans.* £135 7*s.* 4*d.*
2. 29 sheep at £1 17*s.* 2*d.* each. *Ans.* £53 17s. 10*d.*

3. 47 chairs at 18*s.* 6*d.* each. *Ans.* £43 9*s.* 6*d.*
4. 19 lambs at 14*s.* 4*d.* each. *Ans.* £13 12*s.* 4*d.*
5. 37 books at 15*s.* 8*d.* each. *Ans.* £28 19*s.* 8*d.*
6. If a man earn 3*s.* 6*d.* a day, what will be his wages for 17 days? *Ans.* £2 19*s.* 6*d.*
7. 41 lbs. of coffee at 2*s.* 4*d.* per lb. *Ans.* £4 15*s.* 8*d.*
8. 59 cwts. of coals at 1*s.* 7½ per cwt. *Ans.* £4 15*s.* 10½*d.*
9. 92 cwts. of sugar at £2 17*s.* per cwt. *Ans.* £262 4*s.* 0*d.*
10. 53 chests of tea at £12 8*s.* 6*d.* per chest. *Ans.* £658 10*s.* 6*d.*

(1.) 3459654 × 19	(2.) 1479852 × 43	(3.) 4329567 × 23
(4.) 6029397 × 17	(5.) 239976 × 127	(6.) 308858 × 234
(7.) 136653 × 306	(8.) 154261 × 50	(9.) 256641 × 312

Answers.

(1.) 65733426	(2.) 63633636	(3.) 99580041
(4.) 102499749	(5.) 30476952	(6.) 72272772
(7.) 41815818	(8.) 7713050	(9.) 80071992

	£	s.	d.				£	s.	d.		
(1.)	23	2	4	×	17	(2.)	20	16	4	×	23
(3.)	54	16	5¼	×	19	(4.)	17	5	0¼	×	37
(5.)	25	18	2½	×	39	(6.)	26	17	8½	×	47
(7.)	35	19	3	×	57	(8.)	62	10	11	×	34
(9.)	10	17	11¼	×	13	(10.)	15	14	8½	×	68

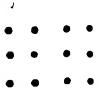
Answers.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	392	19	8	(2.)	478	15	8	(3.)	1041	12	3¾
(4.)	638	5	9¼	(5.)	1010	10	1½	(6.)	1263	12	3½
(7.)	2049	17	3	(8.)	2126	11	2	(9.)	141	13	2¼
(10.)	1070	0	2								

DIVISION.

22. Division consists in finding the number of times that one number is contained in, or can be taken out of another number; thus, 30 divided by 5 means the number of times that 5 can be taken out of 30; the operation is indicated by $30 \div 5$, or $\frac{30}{5} = 6$.

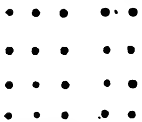
23. The fourth of 12 is the same thing as finding how many fours are contained in 12. Thus the 4th of $12 = 3$, and $12 \div 4 = 3$.



In order to prove this axiom let 12 dots or counters be arranged in rows of fours, then the 4th of 12 will be the number in one column, that is 3, and at the same time it will be seen that the 4 counters, in each row, are repeated 3 times to make up 12, that is 4 is contained in or can be taken out of 12 three times.

24. If any number be divided into two or more groups of units, then the collected units will contain the divisor as often as it is contained in the parts; thus,

$$20 = 12 + 8, \therefore \frac{20}{4} = \frac{12}{4} + \frac{8}{4}.$$



To prove this axiom, let 20 counters, or dots, be arranged as in the annexed figure: here we first observe that 4 can be taken out of 20 five times: in the group to the left there are 12 counters, out of which 4 can be taken 3 times, and in the group to the right there are 8 counters, out of which 4 can be taken 2 times; that is, 4 will be contained in 20 the same number of times that it is contained in 12, together with the number of times it is contained in 8. It is on this principle that we perform operations of division.

25. To divide 9436 by 4.

4)9436
——
2359

Here the 4th of 9 thousands is 2 thousands, with 1 thousand or 10 hundreds as a remainder, then putting these 10 hundreds to

the 4 hundreds we have the 4th of 14 hundreds is 3 hundreds with 2 hundreds or 20 tens as a remainder, then putting these 20 tens to the 3 tens we have the 4th of 23 tens is 5 tens with 3 tens or 30 as a remainder, then putting these 30 units to the 6 units we have the 4th of 36 is 9.

Or thus, 4 is contained in 9, 2 times with 1 as a remainder, that is, 4 is contained in 9 thousands 2 thousand times, with 1 thousand or 10 hundreds as a remainder; we now have to find how often 4 can be taken out of 14 hundreds; we then say 4 is contained in 14, 3 times, with 2 as a remainder, that is, 4 is contained in 14 hundreds, 3 hundred times, with 2 hundreds as a remainder, and so on.

Such questions as the following may be put in the course of the demonstration.

Teacher. What is here done with the remainder of thousands?

Pupil. We bring it to hundreds in order to put it to the hundreds in the dividend.

Teacher. How is this done?

Pupil. By counting 10 hundreds for every thousand.

Teacher. What use have we made of the axiom just proved (24)?

Pupil. We first take the 4th part of the thousands, then of the hundreds, and so on for the 4th part of the whole dividend.

26. To find the 5th of £62 8*s.* 4*d.*

£	*s.*	*d*
5)62	8	4
12	9	8

Here the 5th of £62 is £12 with £2 as a remainder, which put into shillings and added to the 8*s.* give 48*s.*, then the 5th of 48*s.* is 9*s.* with 3*s.* as a remainder, these 3*s.* put into pence and added to the 4*d.* give 40*d.*, and then the 5th of 40*d.* is 8*d.*

Examples in short Division.

1. Divide 342 marbles equally amongst 3 boys?

Ans. 114.

2. 4 chests of oranges contain 928 oranges; how many are there in each chest? *Ans.* 232.

3. There were 135 boys in a school; into how many rows of threes can they be put? *Ans.* 45.

4. How many times must I hold up 4 fingers to count 124? *Ans.* 31 *times.*

5. How many pence can I get out of 136 farthings? *Ans.* 34*d.*

6. How many shillings are there in 288 pence? *Ans.* 24*s.*

7. How many shillings are there in 375 pence? *Ans.* 31*s.* 3*d.*

8. If a man walk 4 miles an hour; how many hours will he take to walk 376 miles? *Ans.* 94 *hours.*

9. 8 yards of cloth cost 136 shillings, how much is that a yard? *Ans.* 17*s.*

10. How many lambs at 9 shillings each can be purchased with 198 shillings? *Ans.* 22.

11. If 3 cwts. of cheese cost £7 5*s.* 6*d.*, how much is that for each cwt.? *Ans.* £2 8*s.* 6*d.*

12. If 5 articles cost £8 6*s.* 3*d.*, required the cost of one? *Ans.* £1 13*s.* 3*d.*

13. How many lbs. of butter at 9*d.* per lb. can be purchased for 40*s.* 6*d.*? *Ans.* 54 *lbs.*

14. A man received £2 6*s.* 6*d.* for working 9 days, how much did he receive per day? *Ans.* 5*s.* 2*d.*

15. If a man can build a wall in 288 days, how many days will it take 6 men to build it? *Ans.* 48.

16. 4 calves cost £11 5*s.* 11*d.*; what is the price of each calf? *Ans.* £2 16*s.* $5\frac{3}{4}$*d.*

(1.) $1346 \div 2$	(2.) $5643 \div 3$	(3.) $50568 \div 4$
(4.) $905 \div 5$	(5.) $7404 \div 6$	(6.) $18543 \div 7$
(7.) $6864 \div 8$	(8.) $6704 \div 8$	(9.) $47385 \div 9$
(10.) $260 \div 10$	(11.) $5049 \div 11$	(12.) $67428 \div 12$

Answers.

(1.) 673	(2.) 1881	(3.) 12642	(4.) 181
(5.) 1234	(6.) 2649	(7.) 858	(8.) 838
(9.) 5265	(10.) 26	(11.) 459	(12.) 5619

	£	s.	d.			£	s.	d.	
(1.)	34	13	$4\frac{1}{2}$	$\div$ 2	(2.)	24	5	3	$\div$ 3
(3.)	25	4	8	$\div$ 4	(4.)	74	14	2	$\div$ 5
(5.)	26	16	9	$\div$ 6	(6.)	86	3	9	$\div$ 7
(7.)	98	2	2	$\div$ 8	(8.)	82	17	$8\frac{1}{4}$	$\div$ 9
(9.)	45	15	$7\frac{3}{4}$	$\div$ 10	(10.)	56	5	$6\frac{1}{2}$	$\div$ 11
(11.)	69	8	$5\frac{1}{2}$	$\div$ 11	(12.)	98	13	6	$\div$ 12

Answers.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	17	6	$8\frac{1}{4}$	(2.)	8	1	9	(3.)	6	6	2
(4.)	14	18	10	(5.)	4	9	$5\frac{1}{2}$	(6.)	12	6	3
(7.)	12	5	$3\frac{1}{4}$	(8.)	9	4	$2\frac{1}{4}$	(9.)	4	11	$6\frac{3}{4}$
(10.)	5	2	$3\frac{3}{4}$	(11.)	6	6	$2\frac{1}{2}$	(12.)	8	4	$5\frac{1}{2}$

---

27. When the divisor is greater than 12, the division is usually performed by the method of long division.

To divide 81864 by 24.

```
24)81864(3000 + 400 + 10 + 1, or 3411
   72000 = 3000 times 24
   -----
    9864
    9600 = 400 times 24
    ----
     264
     240 = 10 times 24
     ---
      24
      24 = 1ce 24
      --
      ••
```

Here 24 is contained in 81, 3 times, that is, 24 is contained in 81 thousands 3 thousand times, with 9 thousands as a remainder: we now have to find how often 24 can be taken out of 9864: we then say 24 is contained in 98, 4 times, that is, 24 is contained in 98 hundreds 4 hundred times, with 2 hundreds as a remainder: we now have to find how often 24 can be taken out of 264 we: then say 24 is contained in 26, 1 time, that is, 24 is contained in 26 tens 10 times, with 2 tens as a remainder: we now have to find how often 24 can be taken out of 24, which is 1ce without any remainder.

Suppressing the unnecessary figures, the foregoing process is equivalent to the following rule: 24 cannot be taken out of 8, but taking the two first figures, 24 is contained in 81, 3 times, with 9 as a remainder; bringing down the next figure 8, we find that 24 is contained in 98, 4 times, with 2 as a remainder; bringing down the next figure 6, and so on until all the figures of the dividend are exhausted.

To divide £34 9*s.* 8*d.* by 16.

$$
\begin{array}{rlll}
 & \pounds & s. & d. \\
16) & 34 & 9 & 8\,(\pounds 2\ 3s.\ 1\tfrac{1}{4}d. \\
 & \underline{32} & & \\
 & 2 & & \\
 & \underline{20} & & \\
16) & 49\,(3s. & & \\
 & \underline{48} & & \\
 & 1 & & \\
 & \underline{12} & & \\
16) & 20\,(1d. & & \\
 & \underline{16} & & \\
 & 4 & & \\
 & \underline{4} & & \\
16) & 16\,(\tfrac{1}{4} & & \\
 & \underline{16} & & \\
 & \cdot\cdot & &
\end{array}
$$

In this example we find, first, the sixteenth part of £34 to be £2, with £2 as a remainder, which brought to shillings and added to the 9*s.* gives 49*s.*, then the sixteenth part of 49*s.* is 3*s.* with 1*s.* as a remainder, which brought to pence and added to the 8*d.* gives 20*d.*, then the sixteenth part of 20*d.* is 1*d.* with 4*d.* as a remainder, which brought to farthings gives 16 farthings, then the sixteenth part of 16 farthings is 1 farthing.

28. Questions in long division may be solved by short division, when the divisor is formed of two or more factors, each of which is less than 13.

Taking the two preceding examples, we have by successive division,

$$24 = \left\{\begin{array}{l} 4)\underline{81864} \\ 6)\underline{20466} \\ \quad 3411 \end{array}\right. \qquad 16 = \left\{\begin{array}{rrrr} 4) & 34 & 9 & 8 \\ \hline 4) & 8 & 12 & 5 \\ \hline & 2\ ,, & 3\ ,, & 1\tfrac{1}{4} \end{array}\right.$$

These operations depend upon the axiom, that we may divide one number by another, by successively dividing by the factors of the divisor: thus, in the annexed figure, we observe,

• • •   • • • • • • • • •   first, that the 12th part of 24 is 2;
• • •   • • • • • • • • •   but the 4th part of the 24 counters is the number contained in each of the four groups, and then the 3rd part of the number in each one of them is 2.

In order to give another illustration of this important principle, let it be required to find the cost of 1 article, when 15 cost £4 16*s.* 3*d.*

$$\begin{array}{rrrrl} & £ & s. & d. & \\ 3\,) & 4 & 16 & 3 & = \text{cost of 15 articles.} \\ \hline 5\,) & 1 & 12 & 1 & = \text{cost of \ 5 articles.} \\ \hline & & 6 & 5 & = \text{cost of \ 1 article.} \end{array}$$

Here, because $3 \times 5 = 15$, we successively divide by 3 and 5 to find the cost of 1 article; because the cost of 5 articles will be the 3rd part of the cost of 15 articles, and then the cost of 1 article will be the 5th part of the cost of 5 articles.

29. It is useful to observe, that when we divide any number by 10, 100, &c., we simply cut off as many figures from the right hand place of the dividend as there are ciphers in the divisor, then the figures thus cut off are the remainder, and the other figures are the quotient. Thus to divide 3642 by 10, and 48325 by 100, we have

1,0) 364,2	1,00) 483,25
364,—2 *rem.*	483,— 25 *rem.*

In like manner to divide by tens or hundreds, we have

5,0) 3478,9	4,00) 654,23
695,— 39 *rem.*	163,— 223 *rem.*

30. Useful application. In 375 shillings how many pounds are there?

2,0) 37,5
£18 15*s.*

Here, in dividing by 20, we first cut off the units' figure, which gives a remainder of 5 after dividing by 10, then dividing by 2, we have a remainder of 1 ten from the 7 tens; this remainder put to the 5, gives a total remainder of 15*s.*

Examples in Division.

1. There are 416 apples in 13 baskets; how many apples does each contain? *Ans.* 32.

2. Divide 228 marbles equally among 19 boys; how many will each receive? *Ans.* 12.

3. A farmer has 232 sheep, and he wishes to put them into 29 equal lots; how many sheep should he have in each lot? *Ans.* 8.

4. I paid 16*s.* 4*d.* for 14 lbs. of coffee; how much is that a lb.? *Ans.* 1*s.* 2*d.*

5. If 53 lbs. of butter cost £2 17*s.* 5*d.*, what is the price of 1 lb.? *Ans.* 1*s.* 1*d.*

6. If a man walk 25 miles every day, in what time will he walk 325 miles? *Ans.* 13 days.

7. If 1 yard of cloth can be bought for 15*s.*, how many yards can be had for 180*s.*? *Ans.* 12.

8. If 30 eggs cost 6*s.* 3*d.*, what is that a piece? *Ans.* 2½*d.*

9. Divide 153 pence equally among 17 persons. *Ans.* 9*d.*

10. A grocer sold 34 lbs. of sugar for 238 pence; what was that a lb.? *Ans.* 7*d.*

11. If 63 pairs of shoes cost £50 8*s.*, how much did each pair cost? *Ans.* 16*s.*

12. Bought 78 yards of cloth for £46 6*s.* 3*d.*; what was that for each yard? *Ans.* 11*s.* 10½*d.*

13. How many yards of cloth at 17*s.* per yard can I purchase with £7 13*s.*? *Ans.* 9 *yds.*

14. If 28 pairs of gloves cost £1 18*s.* 6*d.*, how much is that a pair? *Ans.* 1*s.* 4½*d.*

15. Bought 36 gallons of wine for £33 15*s.* 0*d.*; what did it cost a gallon? *Ans.* 18*s.* 9*d.*

16. If a workman receive £7 for 28 days, how much is that per day? *Ans.* 5*s.*

17. If 140 lbs. of tea cost £64 15*s.*, what is the price of each lb.? *Ans.* 9*s.* 3*d.*

18. In 360 shillings how many pounds? *Ans.* £18.

19. In 462*s.* how many pounds? *Ans.* £23 2*s.*

20. How many pounds are there in 364*s.*, 573*s.*, 450*s.*, 589*s.*, 632*s.*, and 210*s.*? *Ans.* £18 4*s.*, £28 13*s.*, £22 10*s.*, £29 9*s.*, £31 12*s.*, and £10 10*s.*

21. Divide £142 6*s.* 8*d.* by 20. *Ans.* £7 2*s.* 4*d.*

(1.)	839160 ÷ 24	(2.)	303696 ÷ 18
(3.)	643356 ÷ 23	(4.)	4733490 ÷ 15
(5.)	15908652 ÷ 18	(6.)	20648 ÷ 29
(7.)	607392 ÷ 72	(8.)	321678 ÷ 46
(9.)	82592 ÷ 58	(10.)	736263 ÷ 67
(11.)	294052 ÷ 134	(12.)	401598 ÷ 201
(13.)	803196 ÷ 804	(14.)	1606392 ÷ 402
(15.)	360048 ÷ 87		

Answers.

(1.)	34965	(2.)	16872	(3.)	27972	(4.)	315566
(5.)	883814	(6.)	712	(7.)	8436	(8.)	6993
(9.)	1424	(10.)	10989	(11.)	2194	(12.)	1998
(13.)	999	(14.)	3996	(15.)	4138.		

	£	s.	d.				£	s.	d.		
(1.)	8	5	9	÷	17	(2.)	4	15	7	÷	31
(3.)	160	6	8	÷	37	(4.)	126	14	0	÷	21
(5.)	63	7	0	÷	42	(6.)	28	13	6	÷	62
(7.)	652	16	2	÷	19	(8.)	326	8	1	÷	38
(9.)	1222	16	0	÷	48	(10.)	670	8	9	÷	400
(11.)	1244	10	10	÷	140	(12.)	1010	10	1½	÷	78
(13.)	4099	14	6	÷	228	(14.)	5208	1	6¾	÷	190

Answers.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	0	9	9	(2.)	0	3	1	(3.)	4	6	8
(4.)	6	0	8	(5.)	1	10	2	(6.)	0	9	3
(7.)	34	7	2	(8.)	8	11	9½	(9.)	25	9	6
(10.)	1	13	6¼	(11.)	8	17	9½	(12.)	12	19	1¼
(13.)	17	19	7½	(14.)	27	8	2½				

ARITHMETICAL TABLES.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

MONEY.

4 Farthings (*f*)	make	1 Penny (*d*)
12 Pence	- - - - -	1 Shilling (*s*)
20 Shillings	- - - - -	1 Pound or Sovereign (£)
5 Shillings	- - - - -	1 Crown
21 Shillings	- - - - -	1 Guinea
1 Farthing is	written	$\frac{1}{4}$
2 Farthings are	- - - -	$\frac{1}{2}$ Or 1 Half-penny
3 Farthings	- - - -	$\frac{3}{4}$

AVOIRDUPOIS WEIGHT.

For all Common Goods.

16 Drams	make	1 Ounce (*oz.*)
16 Ounces	. .	1 Pound (*lb.*)
14 Pounds	. .	1 Stone (*st.*)
28 Pounds	. .	1 Quarter (*qr.*)
4 Quarters (112 lbs.)		1 Hundredweight (*cwt.*)
20 Hundredweight		1 Ton

TROY WEIGHT.

For Gold, Silver and Jewellery.

24 Grains	make	1 Pennyweight (*dwt.*)
20 Pennyweights	. .	1 Ounce
12 Ounces	. . . . .	1 Pound
7000 Grains	. . . . .	1 Pound *Av.*

APOTHECARIES' WEIGHT.

For Drugs, and in Philosophical Experiments.

20 Grains	make	1 Scruple
3 Scruples	. . . . . .	1 Dram
8 Drams	. . . . . .	1 Ounce
12 Ounces	. . . . . .	1 Pound

The gr. oz. and lb. are the same as in Troy Weight.

LONG MEASURE.

4 Inches	make	1 Hand	
12 Inches	. .	1 Foot (*ft.*)	
3 Feet	. .	1 Yard (*yd.*)	
6 Feet	. .	1 Fathom	
5½ Yards	. .	1 Rod, Pole, or Perch	
40 Poles (220 yds)	.	1 Furlong	
8 Furlongs (1760 yds)		1 Mile	
3 Miles	. .	1 League	
2¼ Inches	. .	1 Nail	*Cloth M.*
4 Nails	. .	1 Quarter	*Cloth M.*
4 Quarters	. .	1 Yard	*Cloth M.*
5 Quarters	. .	1 Ell	*Cloth M.*

SQUARE MEASURE.

144 Square Inches	make	1 Square Foot
9 Square Feet	. .	1 Square Yard
30¼ Square Yards	. .	1 Sq. Rod, Pole or Perch (*P.*)
40 Perches	. .	1 Rood (*R.*)
4 Roods (4840 Sq. Yds.)		1 Acre (*Ac.*)
640 Acres	. . . . . .	1 Square Mile

SOLID MEASURE.

1728 Cubic Inches	make	1 Cubic Foot
27 Cubic Feet	. .	1 Cubic Yard

MEASURE OF CAPACITY.

4 Gills or Noggins	make	1 Pint	
2 Pints	. . . .	1 Quart	
2 Quarts	. . . .	1 Pottle	
4 Quarts	. . . .	1 Gallon	
2 Gallons	. . . .	1 Peck	
4 Pecks	. . . .	1 Bushel	
8 Bushels	. . . .	1 Quarter	
5 Quarters	. . . .	1 Load	
3 Bushels	. . . .	1 Sack	*Coal Meas.*
12 Sacks	. . . .	1 Chaldron	*Coal Meas.*
A Barrel of *Beer*	contains	36 Gallons	
A Hogshead	. .	54 Gallons	
A Hogshead of *Wine*	.	63 Gallons	
A Pipe	. . . .	2 Hogsheads	

TIME.

60 Seconds	make	1 Minute
60 Minutes	. .	1 Hour
24 Hours	. .	1 Day
7 Days	. .	1 Week
4 Weeks	. .	1 Lunar Month
365 Days	. .	1 Year

52 Weeks, or 13 Lunar Months, are sometimes reckoned as a Year.

The Year is also divided into 12 Calendar Months, viz.: —

January	31	May	31	September	30
February	28	June	30	October	31
March	31	July	31	November	30
April	30	August	31	December	31

February has 28 Days excepting in Leap Year, which takes place every Fourth Year, and then February has 29 Days. All the other Months contain 31 Days, excepting those named in the Rhyme,

Thirty Days have September,
April, June, and November.

REDUCTION.

31. The operations required in this rule are either effected by multiplication or division.

Ex. 1. Reduce 4 yds. 2 ft. 5 in. to inches.

```
yds. ft. in.
 4    2   5
 3
14 ft.
12
173 inches.
```

Here we first bring the yards to feet by multiplying by 3, adding in the 2 ft.; and then these feet are brought to inches by multiplying by 12, adding in the 5 inches.

Ex. 2. Reduce 863 farthings to shillings.

```
 4)863
12)215d. 3f.
    17s. 11¾d.
```

Here after dividing by 4 to bring the farthings to pence, we have 3*f.* over, then after dividing by 12 to bring the pence to shillings, we have 11*d.* over, so that the given number of farthings contains 17*s.* and 11¾*d.*

Ex. 3. Reduce £12 5*s.* 2¾*d.* to farthings.

```
£12 5s. 2¾d.
   20
  245s.
   12
 2942d.
    4
11771f.
```

Here we first bring the pounds to shillings by multiplying by 20, adding in the 5*s.*; these shillings are brought to pence by multiplying by 12, taking care to add in the 2*d.*; and then these pence are brought to farthings by multiplying by 4, adding in the 3 farthings.

Ex. 4. Reduce 267 days to weeks.

```
7)267 da.
 4)38 w. 1 da.
    9 mo. 2 w. 1 da.
```

Here dividing by 7 to bring the days to weeks, we have 1 day over, then dividing by 4 to bring the weeks to months, we have 2 w. over, so that the given number of days contains 9 mo. 2 w. 1 da.

Examples in Reduction.

1. How many pence are there in £23? *Ans.* 5520*d.*
2. Reduce £14 10*s.* 8¼*d.* to farthings. *Ans.* 13953*f.*
3. How many sixpences are there in £5? *Ans.* 200.
4. How many pence are there in 22 guineas? *Ans.* 5544*d.*
5. In 3 tons 2 cwt. 1 qr., how many lbs.? *Ans.* 6972 *lbs.*
6. In 3 cwt. 2 qrs. 17 lbs., how many lbs.? *Ans.* 409 *lbs.*
7. In 46 tons, how many lbs.? *Ans.* 103040 *lbs.*
8. Reduce 2 qrs. 3 lbs. 4 oz. to drams. *Ans.* 15168 *drams.*
9. Reduce 9 lbs. 5 oz. Troy to grains. *Ans.* 54240 *gr.*
10. Reduce 2 oz. 4 dwts. 5 grs. to grains. *Ans.* 1061 *grs.*
11. In 3 miles 5 yds. 2 ft., how many feet? *Ans.* 15857 *ft.*
12. Reduce 2 yds. 1 ft. 9 in. to inches. *Ans.* 93 *inches.*
13. In 240 fathoms, how many feet? *Ans.* 1440 *ft.*
14. In 40 miles 30 yds., how many feet? *Ans.* 211290 *ft.*
15. Reduce 4 yds. 2 qrs. to nails. *Ans.* 72 *nails.*
16. Reduce 5 ac. 3 r. 27 p. to poles. *Ans.* 947 *poles.*
17. In 9 sq. yds. 7 ft., how many sq. inches? *Ans.* 12672.
18. In 8 cubic yds., how many cubic inches? *Ans.* 373248.
19. Reduce 24 gals. 3 qts. to gills. *Ans.* 792 *gills.*
20. In 40 bushels, how many quarts? *Ans.* 1280 *quarts.*
21. Reduce 2 qrs. 3 pks. to pints. *Ans.* 1072 *pints.*
22. In 2 years 5 m. 3 w., how many minutes? *Ans.* 1280160.
23. If a man live 3 score years and 10, how many seconds does he live? *Ans.* 2201472000 *sec.*
24. Reduce 374 inches to feet. *Ans.* 31 *ft.* 2 *in.*
25. „ 273 qrs. to cwts. *Ans.* 68 *cwts.* 1 *qr.*
26. „ 376 lbs. to cwts. *Ans.* 3 *cwts.* 1 *qr.* 12 *lbs.*
27. „ 157 day to months. *Ans.* 5 *mo.* 2 *w.* 3 *da.*
28. „ 5432 farthings to pounds. *Ans.* £5 13*s.* 2*d.*

29. How many pounds are there in 64 crowns? *Ans.* £16.
30. How many pounds are there in 40 guineas? *Ans.* £42.
31. Reduce 456 sixpences to pounds. *Ans.* £11 8*s.*
32. „ 234 sq. feet to sq. yards. *Ans.* 26 *sq. yds.*
33. „ 23460 lbs. to tons. *Ans.* 10 *tons,* 9 *cwts.* 1 *qr.* 24 *lbs.*
34. „ 6452 oz. to cwts. *Ans.* 3 *cwts.* 2 *qrs.* 11 *lbs.* 4 *oz.*
35. „ 3456 grs. to oz. Troy. *Ans.* 7 *oz.* 4 *dwts.*
36. „ 39460 ft. to miles. *Ans.* 7 *m.* 833 *yds.* 1 *ft.*
37. „ 3492 nails to yards. *Ans.* 218 *yds.* 1 *qr.*
38. „ 564932 inches to leagues. *Ans.* 2 *lea.* 2 *m.* 1612 *yds.* 1 *ft.* 8 *in.*
39. „ 24567 poles to acres. *Ans.* 153 *ac.* 2 *r.* 7 *p.*
40. „ 4642 sq. inches to sq. yds. *Ans.* 3 *yds.* 5 *ft.* 34 *sq. in.*
41. „ 34500 cubic inches to cubic feet. *Ans.* 19 *ft.* 1668 *in.*
42. „ 244 pints to gallons. *Ans.* 30 *gals.* 2 *qts.*
43. „ 4569 days to years. *Ans.* 12 *years,* 7 *mo.* 5 *da.*
44. „ 64598 seconds to hours. *Ans.* 17 *h.* 56 *min.* 38 *sec.*

WEIGHTS AND MEASURES.

Addition.

1. A grocer sold 5 lbs. 12 oz. of bacon to one person, and 8 lbs. 6 oz. to another; how much was sold altogether? *Ans.* 14 *lb.* 2 *oz.*

2. A draper bought 3 pieces of cloth; in the 1st there were 26 yds. 3 qrs. 2 nls., in the 2nd 19 yds. 2 qrs. 3 nls., and in the 3rd 24 yds. 0 qrs. 1 nl.; what quantity did he purchase? *Ans.* 70 *yds.* 2 *qrs.* 2 *nls.*

3. A man went a journey. On the first day he travelled 27 miles 5 fur., on the second day 23 miles 4 fur., and on the third day 21 miles 6 fur.; what distance did he travel?

Ans. 72 *miles* 7 *furlongs.*

4. A schoolmaster took 1 h. 10 min. in giving a Bible lesson, 57 min. in giving a lesson on grammar, and 1 h. 5 min. in giving a lesson on mechanics; what time did he take altogether? *Ans.* 3 *h.* 12 *min.*

	lbs.	*oz.*		*cwt.*	*qr.*	*lb.*		*qr.*	*lbs.*	*oz.*		*lb.*	*oz.*	*dwts.*
(5.)	7	14	(6.)	3	2	16	(7.)	4	8	6	(8.)	3	15	9
	4	12		2	1	8		5	14	9		2	14	8
	6	3		1	6	15		7	9	8		5	4	7

	dwt.	*gr.*		*oz.*	*dwt.*	*gr.*		*oz.*	*dr.*	*scr.*		*dr.*	*scr.*	*gr.*
(9.)	8	9	(10.)	3	8	12	(11.)	9	6	1	(12.)	7	2	9
	2	8		1	9	22		4	5	2		8	1	7
	1	9		2	7	15		7	7	2		1	2	8

	yds.	*ft.*		*lea.*	*m.*	*fur.*		*yds.*	*ft.*	*in.*		*yds.*	*qrs.*	*na*
(13.)	9	2	(14.)	8	1	7	(15.)	5	2	8	(16.)	7	2	3
	5	1		5	2	5		7	1	9		5	3	2
	2	2		1	0	6		3	0	7		4	1	3
	1	0		9	2	4		2	2	6		2	0	0

	ells.	*qrs.*		*sq.yds.*	*sq.ft.*		*ac.*	*r.*	*po.*		*gal.*	*qts.*	*pt.*
(17.)	8	2	(18.)	19	8	(19.)	5	2	31	(20.)	7	2	1
	5	4		6	5		2	3	17		5	3	0
	3	3		15	7		4	1	1		8	1	1

	qrs.	*bus.*		*w.*	*d.*	*hrs.*		*mo.*	*w.*	*d.*		*yrs.*	*d.*
(21.)	5	3	(22.)	3	6	20	(23.)	4	3	4	(24.)	2	200
	2	2		2	4	23		2	2	6		3	324
	3	6		2	5	9		1	3	5		2	262

Answers.

(5.) 18 lb. 13 oz.	(6). 8 cwt. 2 qrs. 11 lb.
(7.) 17 qr. 4 lb. 7 oz.	(8.) 12 lbs. 10 oz. 4 dwts.
(9.) 12 dwt. 2 gr.	(10.) 7 oz. 6 dwt. 1 gr.
(11.) 22 oz. 3 dr. 2 scr.	(12.) 18 dr. 0 scr. 4 gr.
(13.) 18 yds. 2 ft.	(14.) 25 lea. 1 m. 6 fur.
(15.) 19 yds. 1 ft. 6 in.	(16.) 20 yds.
(17.) 17 ells 4 qrs.	(18.) 42 sq. yds. 2 sq. ft.
(19.) 12 ac. 3 r. 9 p.	(20.) 21 gal. 3 qts.
(21.) 11 qrs. 3 bus.	(22.) 9 w. 3 d. 4 hrs.
(23.) 9 mo. 2 w. 1 da.	(24.) 9 yrs. 56 da.

Subtraction.

1. A grocer bought 26 lbs. 6 oz. of tea, and sold 18 lbs. 14 oz.; how much had he left? *Ans.* 7 *lbs.* 8 *oz.*

2. A piece of cloth contained 24 yds. 2 qrs. 3 nls., but 19 yds. 3 qrs. 2 nls. were sold, how much was there remaining? *Ans.* 4 *yds.* 3 *qrs.* 1 *nl.*

3. John is 9 years 7 mon. old, and Thomas is 11 years 2 mon.; how much is Thomas older than John?

Ans. 1 *yr.* 7 *mon.*

4. A room is 14 ft. 3 in. long, and 9 ft. 8 in. broad; how much does the length exceed the breadth?

Ans. 4 *ft.* 7 *in.*

5. Subtract 3 cwt. 3 qrs. from 12 cwt. 1 qr.

Ans. 8 *cwt.* 2 *qrs.*

6. " 2 oz. 6 dwt. 9 gr. from 6 oz. 3 dwt.

Ans. 3 *oz.* 16 *dwt.* 15 *gr.*

7. " 4 bus. 3 pks. from 12 bus. 2 pks.

Ans. 7 *bus.* 3 *pks.*

8. " 2 m. 4 fur. from 11 m. 2 fur.

Ans. 8 *m.* 6 *fur.*

9. " 4 sq. yds. 6 sq. ft. from 9 sq. yds. 3 sq. ft.

Ans. 4 *sq. yds.* 6 *sq. ft.*

10. Subtract 5 ac. 2 r. 32 p. from 9 ac.
Ans. 3 *ac.* 1 *r.* 8 *p.*

11. „ 7 yrs. 5 mon. from 12 yrs. 2 mon.
Ans. 4 *yrs.* 9 *mo.*

12. „ 5 d. 9 hrs. 7 min. from 9 d. 5 hrs.
Ans. 3 *da.* 19 *hrs.* 53 *min.*

Multiplication

1. What is the weight of 3 casks of sugar weighing 5 cwt. 2 qrs. 20 lbs. each? *Ans.* 17 *cwt.* 4 *lbs.*

2. What is the weight of 7 chests of soap weighing 3 cwt. 3 qrs. 9 lbs. each? *Ans.* 26 *cwt.* 3 *qrs.* 7 *lb.*

3. What is the weight of 5 silver plates weighing 5 oz. 4 dwts. 9 grs. each? *Ans.* 26 *oz.* 1 *dwt.* 21 *grs.*

4. What is the length of 6 pieces of calico measuring 24 yds. 2 qrs. 3 nls. each? *Ans.* 148 *yds.* 2 *nls.*

(5.)	4 tons 15 cwt.	× 5.	(6.)	5 dwt. 9 grs.	× 25.
(7.)	3 ft. 8 in.	× 16.	(8.)	5 yds. 2 qrs.	× 36.
(9.)	3 qrs. 5 nls.	× 17.	(10.)	5 ac. 2 r.	× 12.
(11.)	6 qts. 1 pt.	× 24.	(12.)	3 min. 9 sec.	× 14.

(13.) 4 yrs. 7 w. × 29.

Answers.

(5.)	23 tons 15 cwt.	(6.)	134 dwt. 9 gr.
(7.)	58 ft. 8 in.	(8.)	198 yds.
(9.)	72 qrs. 1 nl.	(10.)	66 ac.
(11.)	156 qts.	(12.)	44 mins. 6 sec.

(13.) 119 yrs. 47 w.

Division.

1. 3 chests of tea weigh 169 lbs. 5 oz., what is the weight of each chest? *Ans.* 56 *lbs.* 7 *oz.*

2. A family consumes 5 lbs. 4 oz. of cheese in the week, how much is that a day? *Ans.* 12 *oz.*

3. A man walked 85 miles 4 yds. in 7 days, what distance did he walk in each day? *Ans.* 12 *m.* 252 *yds.*

4. What is the fourth part of 2 sacks 2 bus.? *Ans.* 2 *bus.*

5. What is the fifth part of a field containing 12 ac. 3 r. 5 p.? *Ans.* 2 *ac.* 2 *r.* 9 *p.*

6. Divide a chaldron of coals into 9 equal portions. *Ans.* 1 *sac.* 1 *bus.*

7. 18 horses ate 16 qrs. 2 bus. of corn, how much did each horse eat? *Ans.* 7 *bus.* 1 *gal.* 3 *qts.*

8. Divide 53 mon. 4 w. by 5. *Ans.* 10 *mo.* 3 *w.* 1 *da.* 9 *h.* 36 *min.*

RATIOS.

32. Before commencing this rule, the Teacher must explain the "Formation of a Fraction." See **40.**

The ratio of two numbers is their relative magnitude, or the number of times that one number is contained in the other; thus we express the ratio of 7 to 8 by saying that 7 are 7 times the 8th of 8, or that 7 are $\frac{7}{8}$ of 8. This ratio is also sometimes expressed by 7 : 8, which reads 7 is to 8.

Examples in Ratios.

1. What part of 8 is 2? *Ans.* One-fourth, or $\frac{1}{4}$; because if 8 units be divided into four equal lots, || || || ||, the number in one of them is called the one-fourth ($\frac{1}{4}$) of 8, which we perceive is 2.

2. What part of 6 is 4? *Ans.* $\frac{2}{3}$.

6 = || || || Here it will be seen that the 3rd of 6,
4 = || || which is 2, must be repeated 2 times to produce 4, that is, 4 are 2 times the 3rd, or $\frac{2}{3}$, of 6.

3. What part of 9 is 12? *Ans.* $\frac{4}{3}$; because the 3rd of 9, which is 3, must be repeated 4 times to produce 12.

4. What is the ratio of 15 to 12? *Ans.* $\frac{5}{4}$.

12 = ||| ||| ||| ||| Here the 4th of 12, which is
15 = ||| ||| ||| ||| ||| 3, must be repeated 5 times

to make 15, that is, 15 are 5 times the 4th, or $\frac{5}{4}$, of 12.

This ratio is also obviously expressed by $\frac{15}{12}$; therefore $\frac{5}{4}$ and $\frac{15}{12}$ express the same ratio; but the former is the ratio in its least terms.

5. What is the ratio of 20 to 25? *Ans.* $\frac{4}{5}$.

6. What is 3 times the 4th, or $\frac{3}{4}$, of 16? *Ans.* 12; because the 4th of 16 is 4, and therefore 3 times the 4th of 16 = 3 times 4 = 12.

7. What is one-fifth ($\frac{1}{5}$) of £1.? *Ans.* 4*s.*

8. What is $\frac{2}{3}$ of a shilling? *Ans.* 8*d.*

9. What is $\frac{3}{5}$ of £2.? *Ans.* £1. 4*s.*

10. Find respectively the values of $\frac{2}{5}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{2}{3}$, and $\frac{5}{6}$ of a pound. *Ans.* 8*s.*, 15*s.*, 17*s.* 6*d.*, 13*s.* 4*d.*, 16*s.* 8*d.*

11. Show that $\frac{4}{6}$ of a shilling is the same as $\frac{2}{3}$ of a shilling.

12. What part of a shilling is 2*d.*? *Ans.* $\frac{1}{6}$.

13. „ „ „ „ 4*d.*? *Ans.* $\frac{1}{3}$.

14. „ „ „ „ 8*d.*? *Ans.* $\frac{2}{3}$.

15. „ „ „ „ 9*d.*? *Ans.* $\frac{3}{4}$.

16. „ „ of a pound is 5*s.*? *Ans.* $\frac{1}{4}$.

17. „ „ „ „ 4*s.*? *Ans.* $\frac{1}{5}$.

18. „ „ „ „ 12*s.*? *Ans.* $\frac{3}{5}$.

19. What part of a lb. av. is 8 oz.? *Ans.* $\frac{1}{2}$.

20. „ „ „ „ 2 oz.? *Ans.* $\frac{1}{8}$.

21. „ „ „ „ 4 oz.? *Ans.* $\frac{1}{4}$.

22. „ „ „ „ 12 oz.? *Ans.* $\frac{3}{4}$.

RULE OF THREE

(where a knowledge of Fractions is not required).

33. *First Method. By Multiplication.*

1. If 3 books cost 6*s*. 7*d*., what will 12 cost?

Cost 3 books = 6*s*. 7*d*.

∴ „ 12 „ = 4 times 6*s*. 7*d*. = 1*l*. 6*s*. 4*d*.

2. If 5 mugs cost 6*s*. 7¾*d*., what will 20 cost? *Ans*. £1 6*s*. 7*d*.

3. If 4 sheep cost £6 12*s*. 7½*d*., what will 12 cost? *Ans*. £19. 17*s*. 10½*d*.

4. If 7 chairs cost 18*s*. 9*d*., what will be the charge for 42 chairs at the same rate? *Ans*. £5. 12*s*. 6*d*.

5. How much must be paid for 30 cwts. of sugar, when 5 cwts. cost £9 7*s*. 8½*d*.? *Ans*. £56. 6*s*. 3*d*.

6. If a man can earn £2 5*s*. 6*d*. in 6 days, how much will he earn in 42 days? *Ans*. £15 18*s*. 6*d*.

7. If a bricklayer can build 46 yds. of wall in 9 days, how many yards will he build in 63 days? *Ans*. 322.

8. If 9 oxen cost £63 10*s*. 7*d*., what will 45 cost? *Ans*. £317 12*s*. 11*d*.

9. If 5 lbs. of tea cost £1 6*s*. 3*d*., what will 45 lbs. cost? *Ans*. £11 16*s*. 3*d*.

10. If a man can walk 65 miles in 3 days, what distance will he walk in 15 days? *Ans*. 325 miles.

34. *Second Method. By Division.*

1. If 8 lbs. of sugar cost 6*s*. 9*d*., what will 2 lbs. cost?

Cost of 8 lbs. = 6*s*. 9*d*.

∴ „ 2 lbs. = ¼ of 6*s*. 9*d*. = 1*s*. 8¼*d*.

2. If 12 bottles cost £3 13*s*. 9*d*., what should I pay for 4 bottles at the same rate? *Ans*. £1 4*s*. 7*d*.

3. If 32 horses cost £274 8*s*. 4*d*., how much should be paid for 4 horses at the same rate? *Ans*. £34 6*s*. 0½.

4. If 35 lbs. of tea cost £9 7*s.* 6*d.*, how much should be paid for 7 lbs.? *Ans.* £1 17*s.* 6*d.*

5. 30 articles cost £7 17*s.* 8*d.*, required the cost of 10. *Ans.* £2 12*s.* $6\frac{1}{2}$*d.*

6. 48 cost £21 8*s.* 6*d.*, required the cost of 6. *Ans.* £2 13*s.* $6\frac{3}{4}$*d.*

7. 27 cost £6 4*s.* $3\frac{3}{4}$*d.*, required the cost of 9. *Ans.* £2 1*s.* $5\frac{1}{4}$*d.*

8. 36 cost £18 10*s.* 6*d.*, required the cost of 4. *Ans.* £2 1*s.* 2*d.*

9. 72 cost £16 12*s.* 6*d.*, required the cost of 6. *Ans.* £1 7*s.* $8\frac{1}{2}$*d.*

10. 8 cost £17 14*s.* 4*d.*, required the cost of 2. *Ans.* £4 8*s.* 7*d.*

11. The rent of a farm containing 60 acres is £84 10*s.* 9*d.*, what will be the rent of a field containing 5 acres? *Ans.* £7 0*s.* $10\frac{3}{4}$*d.*

12. If a lb. of sugar cost 6*d.*, what is the cost of 4 oz.? *Ans.* $1\frac{1}{2}$*d.*; because 4 oz. is $\frac{1}{4}$ of a lb.; therefore the cost of 4 oz. will be $\frac{1}{4}$ of 6*d.* = $1\frac{1}{2}$*d.*

13. If a lb. of tea cost 5*s.* 4*d.*, what is the cost of 2 oz.? *Ans.* 8*d.*

14. If a man spend £14 3*s.* 4*d.* in 56 days, how much is that for 7 days? *Ans.* £1 15*s.* 5*d.*

35. *Third Method. By Division and Multiplication.*

1. If 3 articles cost 4*s.* 6*d.*, what is the cost of 7?

Cost 3 articles = 4*s.* 6*d.*
∴ Cost 1 article = $\frac{1}{3}$ of 4*s.* 6*d.* = 1*s.* 6*d.*
∴ Cost 7 articles = 7 times 1*s.* 6*d.* = 10*s.* 6*d.*

In the course of the demonstration, questions like the following may be put by the Teacher.

Teacher (writing down, "*Cost* 3 *articles* ="). What is the cost of 3 articles?

Pupil. Four shillings and sixpence.

Teacher (writing down, "Cost 1 article = "). Will the cost of 1 article be more or less than 4*s.* 6*d.*?

Pupil. It will be less.

Teacher. What part of 4*s.* 6*d.* will 1 article be?

Pupil. One-third part.

Teacher. How many articles have we now to find the cost of?

Pupil. 7 articles.

Teacher. Will the cost of 7 articles be more or less than the cost of one?

Pupil. More. The cost of 7 will be 7 times the cost of one, that is, 7 times 1*s.* 6*d.*

2. If 6 lbs. of coffee cost 15*s.*, what is the cost of 7 lbs.? *Ans.* 17*s.* 6*d.*

3. 5 articles cost 1*s.* 3*d.*, required the cost of 9. *Ans.* 2*s.* 3*d.*

4. 7 " " 1*s.* 9*d.*, " " 5. *Ans.* 1*s.* 3*d.*

5. 12 " " 2*s.* 6*d.*, " " 10. *Ans.* 2*s.* 1*d.*

6. 2 " " 5*s.* 3*d.*, " " 5. *Ans.* 13*s.* $1\frac{1}{2}$*d.*

7. 8 " " 25*s.* 6*d.*. " " 7. *Ans.* 22*s.* $3\frac{3}{4}$*d.*

8. 5 cost £18 6*s.* 3*d.*, required the cost of 13. *Ans.* £47 12*s.* 3*d.*

9. 7 " £2 6*s.* 1*d.*, " " 23. *Ans.* £7 11*s.* 5*d.*

10. 9 " £10 13*s.* " " 70. *Ans.* £82 16*s.* 8*d.*

11. 3 " 17*s.* 3*d.*, " " 31. *Ans.* £8 18*s.* 3*d.*

12. A servant's wages is £1 9*s.* 3*d.* for a quarter or 13 weeks, how much should she receive for 5 weeks? *Ans.* 11*s.* 3*d.*

13. If a person walk 20 miles in 5 hours, how far will he walk in 12 hours? *Ans.* 48 *miles.*

14. If 3 candlesticks cost 8*s.* 4*d.*, how many can be bought with 1£ 13*s.* 4*d.*? *Ans.* 12.

Here the money to be expended is exactly 4 times the cost of 3 candlesticks, therefore the number that can be bought will be 4 times 3 = 12.

15. If 5 yards of cloth cost 7*s.* 5*d.*, how many yards can be purchased with £2 11*s.* 11*d.*? *Ans.* 35 *yds.*

16. 5 mugs cost $2s.\ 6d.$, how many can be bought with $3s.\ 6d.$ or $42d.$?

$$\text{Cost 1 mug} = \tfrac{1}{5} \text{ of } 2s.\ 6d. = 6d.$$
$$\therefore \text{No. mugs} = \tfrac{42}{6} = 7 \textit{ Ans.}$$

17. 6 plates cost 5*s.*, how many will 6*s.* 8*d.* buy? *Ans.* 8.

18. 4 lambs cost 2*l.* 16*s.*, how many can be purchased with 4*l.* 18*s.*? *Ans.* 7.

19. A workman receives 25*s.* for 6 days, how many days must he work to earn 2*l.* 5*s.* 10*d.*? *Ans.* 11 *days.*

20. If 16 cost 9*s.* 4*d.*, how much should be paid for 24?

$$\text{Cost } 16 = 9s.\ 4d.$$
$$\therefore \text{Cost } 4 = \tfrac{1}{4} \text{ of } 9s.\ 4d. = 2s.\ 4d.$$
$$\text{and Cost } 24 = 6 \text{ times } 2s.\ 4d. = 14s.$$

21. If 28 cost 16*s.* 4*d.*, what cost 16? *Ans.* 9*s.* 4*d.*

22. I bought 30 oranges for 2*s.* $2\frac{1}{4}d.$, what did I pay per dozen? *Ans.* $10\frac{1}{2}d.$

36. *Fourth Method. By Division and Addition, or by Multiplication, Division, and Addition.*

1. If 8 lbs. of coffee cost 7*s.* 9*d.*, what will 10 lbs. cost?

$$\text{Cost 8 lbs.} = \ldots\ldots\ 7s.\ 9d.$$
$$\therefore \text{Cost 2 lbs.} = \tfrac{1}{4} \text{ of } 7s.\ 9d. = 1s.\ 11\tfrac{1}{4}d.$$
$$\therefore \text{Cost 10 lbs.} = 9s.\ 8\tfrac{1}{4}d.$$

2. If 8 cost £6 4*s.* $8\frac{1}{2}d.$, what will 12 cost? *Ans.* £9 7*s.* $0\frac{3}{4}d.$

3. If 16 cost £7 9*s.* 2*d.*, what will 18 cost? *Ans.* £8 7*s*, $9\frac{3}{4}d.$

4. If 32 cost £5 3*s.* 7*d.*, what will 40 cost? *Ans.* £6 9*s.* $5\frac{3}{4}d.$

5. It 8 cost £3 5*s.* 4*d.*, what will 9 cost? *Ans.* £3 13*s.* 6*d.*

6. If 63 cost 19*s.* 6*d.*, what will 70 cost?

Here 70 = 63 + 7, and 7 is $\frac{1}{9}$ of 63, then we have,

Cost 63 = — — — 19*s.* 6*d.*
Cost 7 = $\frac{1}{9}$ of 19*s.* 6*d.* = 2*s.* 2*d.*
∴ = Cost 70 21*s.* 8*d.*

7. If 12 cost £21 10*s.*, what will 16 cost?
Ans. £28 13*s.* 4*d.*

8. If 48 cost £120 11*s.* 10*d.*, what will 60 cost?
Ans. £150 14*s.* 9½*d.*

9. If 24 cost £33 15*s.* 7*d*, what will 28 cost?
Ans. £39 8*s.* 2*d.*

10. If 9 cost 3*s.* 2¼*d.*, what will 19 cost?
Here 19 = 2 × 9 + 1, therefore we have,

Cost 18 = 2 times 3*s.* 2¼*d.* = 6*s.* 4½*d.*
Cost 1 = $\frac{1}{9}$ of 3*s.* 2¼*d.* = 4¼*d.*
∴ Cost 19 = 6*s.* 8¾*d.*

11. If 4 cost £6 10*s.* 8*d.*, what will 17 cost?
Ans. £27 15*s.* 4*d.*

12. If 9 cost £12 5*s.* 6*d.*, what will 21 cost?
Ans. £28 12*s.* 10*d.*

13. If 16 cost £18 7*s.* 4*d.*, what will 20 cost?
Ans. £22 19*s.* 2*d.*

14. If 8 cost £6 14*s.* 5*d.*, what will 12 cost?
Ans. £10 1*s.* 7½*d.*

15. If 18 cost £16 2*s.* 3*d.*, what will 27 cost?
Ans. £24 3*s.* 4½*d.*

16 If 6 cost 9*s.* 8*d.*, what will 20 cost?

Cost 18 = 3 times 9*s.* 8*d.* = £1 9*s.* 0*d.*
Cost 2 = $\frac{1}{3}$ of 9*s.* 8*d.* = 3*s.* 2½*d.*
∴ Cost 20 = £1 12*s.* 2½*d.*

17. If 7 hats cost 19*s.*, what will 15 cost?
Ans. £2 0*s.* 8½*d.*

18. If 8 cost £4 16*s.* 10*d.*, what will 11 cost?
Ans. £6 13*s.* 1¾*d.*

19. If 7 cost 5*s.* 8*d.*, what will 23 cost? *Ans.* 18*s.* 7½*d.*

20. If 3 lbs. 2 oz. of tea cost £1 5*s.* 5*d.*, what should be paid for 3lbs. 12 oz. at the same rate? *Ans.* £1 10*s.* 6*d.*

37. *Fifth Method. By Division and Subtraction, or by Multiplication, Division, and Subtraction.*

1. If 12 books cost 6*s.* 3*d.*, what should I pay for 9?

Cost 12 books = — — — 6*s.* 3*d.*
Cost 3 books = $\frac{1}{4}$ of 6*s.* 3*d.* = 1*s.* 6$\frac{3}{4}$*d.*

∴ Cost of 9 books = 4*s.* 8$\frac{1}{4}$*d.*

2. If 25 cost £6 10*s.* 5*d.*, what will 20 cost? *Ans.* £5 4*s.* 4*d.*

3. If 32 cost £16 5*s.* 4*d.*, what will 24 cost? *Ans.* £12 4*s.*

4. If 24 cost £6 5*s.* 9*d.*, what will 20 cost? *Ans.* £5 4*s.* 9$\frac{1}{2}$*d.*

5. If 30 cost £22 10*s.* 5*d.*, what will 27 cost? *Ans.* 20 5*s.* 4$\frac{1}{2}$*d.*

6. If 7 lbs. of rice cost 15$\frac{3}{4}$*d.*, what should I pay for 20 lbs.?

Cost 21 lbs. = 3 times 15$\frac{3}{4}$*d.* = 3*s.* 11$\frac{1}{4}$*d.*
Cost 1 lb. = $\frac{1}{7}$ of 15$\frac{3}{4}$*d.* = 2$\frac{1}{4}$*d.*

∴ Cost 20 lbs. = 3*s.* 9*d.*

7. If 4 cost 9*s.* 6*d.*, what will 19 cost? *Ans.* £2 5*s.* 1$\frac{1}{2}$*d.*

8. If 8 cost 16*s.* 4*d.*, what will 23 cost? *Ans.* £2 6*s.* 11$\frac{1}{2}$*d.*

9. If 6 chairs can be bought for 12*s.* 3*d.*, what must be paid for 17 chairs? *Ans.* £1 14*s.* 8$\frac{1}{2}$*d.*

38. *Sixth Method. By Ratios.*

1. What will be the cost of 20 sheep, if 25 cost £20 0*s.* 5*d.*? Here 20 is $\frac{4}{5}$ of 25.

∴ Cost 20 = $\frac{4}{5}$ of £20 0*s.* 5*d.* = £16 0*s.* 4*d.* *Ans.*

2. If 16 cost £21 7*s.* 4*d.*, what will 12 cost? *Ans.* £16 0*s.* 6*d.*

3. If 30 cost £7 17*s.* 8*d.*, what will 10 cost? *Ans.* £2 12*s.* 6$\frac{1}{2}$*d.*

4. What will be the cost of 90 galls. of spirits, if 80 galls. cost £76 7*s.* 4*d.*? *Ans.* £85 18*s.* 3*d.*

5. If 15 loaves cost 2*s.* 1*d.*, what is the price of 9? *Ans.* 1*s.* 3*d.*

6. If 18 lbs. cost 14 guineas, what cost 21 lbs.? *Ans.* £17 3*s.*

BILLS.

39. It is highly desirable that the pupils should be frequently practised as well in calculating mentally, as with the slate, the cost of the different articles in a simple account.

Ex. 1. $4\frac{3}{4}$ lbs. of tea, at 6*s.* 4*d.*

	s.	*d.*	
	6	4	cost 1 lb.
		4	
1	5	4	cost 4 lbs.
	3	2	cost $\frac{1}{2}$ lb.
	1	7	cost $\frac{1}{4}$ lb.
£1	10	1	cost $4\frac{3}{4}$ lbs.

Here the cost of $\frac{1}{2}$ lb. is the $\frac{1}{2}$ of 6*s.* 4*d.*; and the cost of $\frac{1}{4}$ lb. is the $\frac{1}{2}$ of 3*s.* 2*d.*

Ex. 2. 3 lbs. 4 oz. of coffee, at 2*s.* 6*d.*

2	6	
	3	
7	6	cost 3 lbs.
	$7\frac{1}{2}$	cost 4 oz. or $\frac{1}{4}$ of 1 lb.
8	$1\frac{1}{2}$	cost 3lbs. 4 oz.

A DRAPER'S BILL.

July 1. 1845.

Mr. Hunter,

Bought of T. Rate,

			£	s.	d.
3 yds. of Cloth, ...	at 5*s*. 6*d*.	......			
4 yds. „	at 4 3	......			
7 yds. of Calico,.........	at 8½	......			
2½ „ „	at 4	......			
			£1	19	3½

A GROCER'S BILL.

May 9. 1845.

Mr. Thomas Bolton,

Bought of H. Winter,

May 1.	3 lbs. of Tea,............	at 4*s*. 3*d*.	......			
	4 lbs. 2 oz. of Bacon...	at 6	......			
	½ lb. of Coffee	at 2 3	......			
4.	2 lbs. 3 oz. of Soap, ...	at 8	......			
9.	3½ lbs. of Currants, ...	at 7	......			
				£0	19	5¼

FRACTIONS.

40. *Formation of Fractions.* A fraction is formed by dividing a unit, or any object representing a unit, into a certain number of equal parts, then any number of those parts will form a fraction; thus, if I cut a loaf into 5 equal parts, one of those parts will be called one-fifth ($\frac{1}{5}$), two of them will be called two-fifths ($\frac{2}{5}$), three of them three-fifths ($\frac{3}{5}$), and so on. In the fraction $\frac{3}{5}$, the 5 is called the denominator, and the 3

the numerator; the denominator, therefore, indicates the number of parts into which the unit is divided, and the numerator shows how many of those parts are taken.

1. Show how the fraction $\frac{3}{4}$ is formed.

Ans. If I cut an apple into 4 equal pieces, 3 of those pieces will be $\frac{3}{4}$ of the apple. Or thus: if I divide the line A ____|____|____n____ B into 4 equal parts, 3 of them, as A *n*, will be $\frac{3}{4}$ of the line. Or thus: if I divide any space, say a square, into 4 equal parts, 3 of them will be $\frac{3}{4}$ of the whole space.

2. Show how I should give a boy $\frac{2}{5}$ of an apple.

Ans. I should cut the apple into 5 equal pieces, then one of them would be $\frac{1}{5}$ of the apple, and 2 of them would be $\frac{2}{5}$ of the apple. So that if a boy got 2 of the pieces, he would have $\frac{2}{5}$ of the apple.

If I give 2 of the pieces to one boy, and 2 more to another boy, how much of the apple would I have given away?

Ans. $\frac{4}{5}$; thus, $\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$.

3. Show how I should give a person $\frac{3}{7}$ of a loaf.

Ans. I should cut the loaf into 7 equal pieces, then 3 of these pieces would be $\frac{3}{7}$ of the loaf.

If I give 3 pieces to one person, and 2 of them to another, how much of the loaf would I have given away?

Ans. $\frac{5}{7}$; thus, $\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$.

How many sevenths make up the whole or unit?

Ans. Seven; thus, $\frac{7}{7} = 1$.

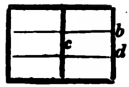
How do you add fractions having the same denominator?

Ans. By adding the numerators together, and then putting the common denominator beneath that sum: thus 3 pieces of a loaf, added to 2 pieces of the same size, will give 5 of those pieces, no matter what part of the loaf each piece may be.

4. Show how the fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, &c., are formed.

41. A fraction may also be regarded as a quantity resulting from the division of the numerator by the denominator; thus the 4th part of 3 is the same as $\frac{3}{4}$; because the 4th of

3 loaves is the same as $\frac{3}{4}$ of one loaf; again, the annexed figure shows that the $\frac{2}{3}$ of one square, or unit, is the same as the third part of 2 squares or units. The line *a b* cuts off the third of 2 units, and *c d* $\frac{2}{3}$ of one unit.



To give another illustration of this important principle: let 3 persons have each the same *sum of money*, then if we take the $\frac{1}{3}$ of each man's money from him, we shall have got 3 times the $\frac{1}{3}$ of each man's money, and at the same time the $\frac{1}{3}$ of the whole money; that is, 3 times $\frac{1}{3}$ of the *sum of money* $=\frac{1}{3}$ of 3 times the *sum of money*.

1. Show that $\frac{3}{5}$ of 1 pound is the same as $\frac{1}{5}$ of 3 pounds. *Ans.* 12*s.*

2. Show that $\frac{3}{4}$ of 1 shilling is the same as $\frac{1}{4}$ of 3 shillings. *Ans.* 9*d.*

3. Show that $\frac{5}{7}$ of 15*s.* 9*d.* is the same as $\frac{1}{7}$ of 5 times 15*s.* 9*d.*

	s.	*d.*
7)	15	9
	2	3
		5
	11	3

	s.	*d.*
	15	9
		5
7)	78	9
	11	3

4. Show that $\frac{7}{8}$ of £9 6*s.* 8*d.* is the same as $\frac{1}{8}$ of 7 times £9 6*s.* 8*d.* *Ans.* £8 3*s.* 4*d.*

It therefore appears, that we can find the fractional part of any quantity in two ways; 1st, by dividing the quantity by the denominator of the fraction, and then multiplying the quotient by the numerator; 2nd, by multiplying the quantity by the numerator of the fraction, and then dividing the product by the denominator. In general the latter method is the more convenient in practice.

42. *To reduce an improper Fraction to a mixed Number, and conversely.*

An *improper* fraction is a fraction whose numerator is

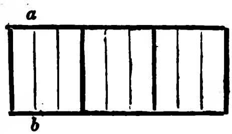
greater than its denominator; as $\frac{5}{3}$, $\frac{8}{7}$. A *mixed* number is formed by a whole number and a fraction; as $3\frac{2}{7}$, $4\frac{5}{9}$.

1. Suppose I cut a lot of apples, each into 5 equal pieces, then each piece will be called $\frac{1}{5}$. Now if I give a boy 7 of these pieces, how many apples will he have got? *Ans.* $1\frac{2}{5}$; because 5 of the pieces will make 1 apple, and the 2 pieces more will be $\frac{2}{5}$ of an apple, that is, $\frac{7}{5}=\frac{5}{5}+\frac{2}{5}=1\frac{2}{5}$.

If I give a boy 14 of these pieces, how many apples will he have got? *Ans.* $2\frac{4}{5}$; because 10 of the pieces will make 2 apples, and the 4 pieces more will be $\frac{4}{5}$ of an apple, that is, $\frac{14}{5}=\frac{5}{5}+\frac{5}{5}+\frac{4}{5}=2\frac{4}{5}$.

What is here done to bring this improper fraction to a mixed number? *Ans.* We find how many fives can be got out of 14, and the number that is over, gives us the number of fifths.

If I give a boy 4 apples and 2 of the pieces ($\frac{2}{5}$) more, how many fifths will he have got? *Ans.* $\frac{22}{5}$, because each apple will cut into 5 fifths, and 4 apples will cut into 4 times 5, or 20 fifths, and the 2 fifths more will make 20 fifths; that is, $4\frac{2}{5}=\frac{5}{5}+\frac{5}{5}+\frac{5}{5}+\frac{5}{5}+\frac{2}{5}=\frac{22}{5}$.



2. The improper fraction $\frac{8}{3}=2\frac{2}{3}$; because $\frac{3}{3}=1$, $\therefore\ \frac{8}{3}=\frac{3}{3}+\frac{3}{3}+\frac{2}{3}=2\frac{2}{3}$. Performing the reverse operation, $2\frac{2}{3}=\frac{8}{3}$; because $2=\frac{6}{3}$, and $\therefore$ $2\frac{2}{3}=\frac{6}{3}+\frac{2}{3}=\frac{8}{3}$. The annexed figure renders these processes apparent, where *a b* cuts off $\frac{8}{3}=2\frac{2}{3}$.

Hence we derive the following *rules*. An improper fraction is brought to a mixed number by dividing the numerator by the denominator for the whole number, and the remainder, if any, will be the numerator of the fractional part. A mixed number is brought to an improper fraction, by multiplying the whole number by the denominator of the fraction, and adding the numerator of the fraction for the new numerator.

Examples.

1. In 9 quarter oranges, how many whole oranges? *Ans.* $2\frac{1}{4}$.

2. How many one-fifths of a loaf can I cut out of 3 loaves and $\frac{2}{5}$? *Ans.* $\frac{17}{5}$.

3. Reduce to mixed numbers, proving the processes by a figure, $\frac{7}{4}$, $\frac{13}{5}$, $\frac{19}{7}$, $\frac{11}{3}$. *Ans.* $1\frac{3}{4}$, $2\frac{3}{5}$, $2\frac{5}{7}$, $3\frac{2}{3}$.

4. Reduce to improper fractions, $3\frac{2}{5}$, $4\frac{1}{2}$, $2\frac{3}{7}$. *Ans.* $\frac{17}{5}$, $\frac{9}{2}$, $\frac{17}{7}$.

5. In $3\frac{1}{2}$, $5\frac{1}{2}$, $19\frac{1}{2}$, 15, how many halves? *Ans.* $\frac{7}{2}$, $\frac{11}{2}$, $\frac{39}{2}$, $\frac{30}{2}$.

6. In $5\frac{2}{3}$, 4, $8\frac{1}{3}$, $4\frac{2}{3}$, how many thirds? *Ans.* $\frac{17}{3}$, $\frac{12}{3}$, $\frac{25}{3}$, $\frac{14}{3}$.

7. In $3\frac{1}{4}$, $5\frac{3}{4}$, 2, $2\frac{1}{4}$, how many fourths? *Ans.* $\frac{13}{4}$, $\frac{23}{4}$, $\frac{8}{4}$, $\frac{9}{4}$.

8. In $6\frac{4}{5}$, $9\frac{1}{5}$, $7\frac{3}{5}$, how many fifths? *Ans.* $\frac{34}{5}$, $\frac{46}{5}$, $\frac{38}{5}$.

9. In $2\frac{3}{10}$, $5\frac{7}{10}$, $4\frac{1}{10}$, how many tenths? *Ans.* $\frac{23}{10}$, $\frac{57}{10}$, $\frac{41}{10}$.

10. Reduce to mixed numbers, $\frac{13}{5}$, $\frac{19}{9}$, $\frac{56}{11}$, $\frac{24}{10}$, $\frac{37}{10}$, $\frac{246}{19}$, $\frac{367}{24}$. *Ans.* $2\frac{3}{5}$, $2\frac{1}{9}$, $5\frac{1}{11}$, $2\frac{4}{10}$, $3\frac{7}{10}$, $12\frac{18}{19}$, $15\frac{7}{24}$.

11. Reduce to improper fractions, $14\frac{3}{7}$, $18\frac{9}{14}$, $44\frac{13}{15}$, $24\frac{3}{10}$, $34\frac{5}{10}$. *Ans.* $\frac{101}{7}$, $\frac{261}{14}$, $\frac{673}{15}$, $\frac{243}{10}$, $\frac{345}{10}$.

43. ***To perform Division when there is a Remainder.***

Ex. $34 \div 5 = \frac{34}{5} = 6\frac{4}{5}$. This shows that in division we put the remainder as the numerator, and the divisor as the denominator of a fraction.

Examples.

1. Divide 45 by 4, 563 by 7, 5847 by 17. *Ans.* $11\frac{1}{4}$, $80\frac{3}{7}$, $343\frac{16}{17}$.
2. Divide 23456 by 29. *Ans.* $808\frac{24}{29}$.
3. Divide 72341 by 39. *Ans.* $1854\frac{35}{39}$.
4. Divide 102087 by 61. *Ans.* $1673\frac{34}{61}$.
5. Divide 205050 by 135. *Ans.* $1518\frac{120}{135}$ *or* $\frac{8}{9}$.
6. Divide 1425609 by 93. *Ans.* $15329\frac{12}{93}$ *or* $\frac{4}{31}$.

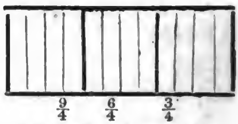
44. *To multiply or divide a Fraction by a whole Number.*

1. If I cut a lot of apples each into 3 equal pieces, then each piece will be $\frac{1}{3}$. Now if I give 2 of these pieces to one boy, 2 pieces to another, 2 pieces to another, and 2 pieces to another, how many times $\frac{2}{3}$ shall I have given away?
Ans. 4 *times* $\frac{2}{3} = \frac{8}{3}$, *that is,* 4 *times* $\frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{8}{3}$.

If to each of the 4 boys I give 5 of the pieces, how many shall I have then given away? *Ans.* 4 *times* $\frac{5}{3} = \frac{20}{3}$, *that is,* $\frac{5}{3} \times 4 = \frac{20}{3}$.

Here it appears, that *we multiply the numerator by the whole number, and retain the denominator.*

2. In the annexed figure it will be seen, that 3 times $\frac{3}{4} = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{9}{4}$ or $2\frac{1}{4}$.



Here also it will be seen, that $\frac{1}{3}$ of $\frac{9}{4} = \frac{3}{4}$, that is $\frac{9}{4} \div 3 = \frac{3}{4}$. Proving, in this case, that *we divide the numerator of the fraction by the whole number.*

3. If to 10 persons I give $\frac{2}{7}$ of a loaf to each, how many loaves shall I have given away? *Ans.* $2\frac{6}{7}$.

What is the $\frac{1}{10}$ of $\frac{20}{7}$, (the loaves given away)? *Ans.* $\frac{2}{7}$.

4. Let *a d b c* (see fig. Art. **46.** page 65.) be any space representing a whole or unit, and let it be divided into 3 equal parts by the upright lines; then the piece *a b c* will be $\frac{1}{3}$; and if we divide this third into 4 equal parts, one of them, as *a c*, will be the fourth of one-third, and at the same time one twelfth of the whole, that is, $\frac{1}{4}$ of $\frac{1}{3}$, or $\frac{1}{3} \div 4 = \frac{1}{12}$. And $\frac{1}{4}$ of $\frac{2}{3}$, or $\frac{2}{3} \div 4 = 2$ times $\frac{1}{12} = \frac{2}{12}$. Here *we multiply the denominator by the divisor.*

5. What is the 5th of $\frac{3}{4}$? *Ans.* $\frac{3}{20}$; because the 5th of $\frac{1}{4} = \frac{1}{20}$, and then the 5th of $\frac{3}{4} = 3$ times $\frac{1}{20} = \frac{3}{20}$.

Examples.

1. Repeat $\frac{1}{2}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{5}{4}$, $\frac{3}{7}$ three times. *Ans.* $1\frac{1}{2}$, $1\frac{1}{5}$, $1\frac{1}{8}$, $3\frac{3}{4}$, $1\frac{2}{7}$.
2. What is the 4th part of $\frac{20}{3}$? *Ans.* $\frac{5}{3}$.
3. Divide $\frac{9}{7}$ into 3 equal portions. *Ans.* $\frac{3}{7}$.
4. What is the 4th part of $\frac{1}{2}$ of an apple? *Ans.* $\frac{1}{8}$.

5. A boy got $\frac{1}{3}$ of an orange, and gave a half of it to his friend, what part of the orange had he left? *Ans.* $\frac{1}{6}$.

(1.) $\frac{7}{2} \times 5$ (2.) $\frac{17}{3} \times 8$ (3.) $\frac{16}{5} \times 9$ (4.) $\frac{9}{10} \times 8$
(5.) $\frac{13}{3} \times 9$ (6.) $\frac{11}{4} \times 19$ (7.) $\frac{34}{5} \times 7$ (8.) $7\frac{1}{8} \times 9$
(9.) $4\frac{5}{7} \times 3$ (10.) $7\frac{2}{9} \times 5$ (11.) $\frac{1}{2}$ of $\frac{8}{9}$ (12.) $\frac{1}{3}$ of $\frac{9}{8}$
(13.) $\frac{1}{2}$ of $\frac{3}{8}$ (14.) $\frac{1}{3}$ of $\frac{2}{7}$ (15.) $\frac{1}{4}$ of $\frac{1}{4}$ (16.) $\frac{1}{3}$ of $\frac{1}{6}$
(17.) $\frac{2}{5} \div 7$ (18.) $\frac{5}{7} \times 12$.

Answers.

(1.) $17\frac{1}{2}$ (2.) $45\frac{1}{3}$ (3.) $28\frac{4}{5}$ (4.) $7\frac{2}{10}$ (5.) 39
(6.) $52\frac{1}{4}$ (7.) $47\frac{3}{5}$ (8.) $64\frac{1}{8}$ (9.) $14\frac{1}{7}$ (10.) $36\frac{1}{9}$
(11.) $\frac{4}{9}$ (12.) $\frac{3}{8}$ (13.) $\frac{3}{16}$ (14.) $\frac{2}{21}$ (15.) $\frac{1}{16}$
(16.) $\frac{1}{18}$ (17.) $\frac{2}{35}$ (18.) $8\frac{4}{7}$.

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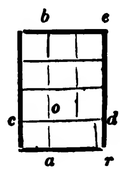
45. It is useful to remember that *a fraction multiplied by its denominator gives the numerator;* thus $\frac{3}{5} \times 5 = \frac{15}{5} = 3$; because $\frac{1}{5}$ taken 5 times gives the whole unit, and $\therefore$ $\frac{3}{5}$ taken 5 times $= 3$ times $1 = 3$.

46. *To change the Denominator of a Fraction.*

1. Bring $\frac{2}{3}$ and $\frac{3}{4}$ to the same denominator, viz. 12ths. Here the space, or unit, is divided into thirds by the upright lines, and into fourths by the horizontal lines. By this means the unit is divided into 12 equal parts, and therefore each part is $\frac{1}{12}$. The line $a\ b$ cuts off $\frac{2}{3}$, and $c\ d$ cuts off $\frac{3}{4}$.



$$\therefore \frac{1}{3} = \frac{4}{12}, \text{ and } \frac{2}{3} = 2 \text{ times } \frac{4}{12} = \frac{8}{12}.$$
$$\frac{1}{4} = \frac{3}{12}, \text{ and } \frac{3}{4} = 3 \text{ times } \frac{3}{12} = \frac{9}{12}.$$

2. How can I cut $\frac{3}{4}$ of a loaf into eighths? *Ans.* By cutting each of the fourths into two equal parts, we shall have, $\frac{1}{4} = \frac{2}{8}$, and $\therefore$ $\frac{3}{4} = 3$ times $\frac{2}{8} = \frac{6}{8}$.

We observe, from the foregoing demonstrations, that if the numerator and denominator of a fraction be at the same time

multiplied or divided by the same number, the value of the fraction will not be altered; thus $\frac{2}{3} = \frac{8}{12}$, where the numerator 2 is multiplied by 4, and the denominator 3 is also multiplied by 4 to obtain $\frac{8}{12}$. By multiplying the denominator by 4 we make the fractional part one-fourth of what it was, but by taking the numerator 4 times we restore the original value of the fraction; the process is, in fact, dividing and multiplying by the same number. The fraction $\frac{2}{3}$ is in its lowest terms, and it is obtained from $\frac{8}{12}$ by dividing numerator and denominator by 4.

3. How many 16ths of a loaf can be cut out of $\frac{3}{4}$ of a loaf? Here, by cutting each $\frac{1}{4}$ into 4 pieces, we shall get $\frac{4}{16}$, and then out of $\frac{3}{4}$ we shall get 3 times $\frac{4}{16} = \frac{12}{16}$.

4. Bring $\frac{1}{2}$ of an apple and $\frac{1}{3}$ of an apple to the same piece or part of an apple. Here, by cutting the $\frac{1}{2}$ into 3 equal pieces, we shall get $\frac{3}{6}$, and, by cutting the $\frac{1}{3}$ into 2 equal pieces, we shall get $\frac{2}{6}$.

5. Show that $\frac{1}{2}$ of an orange is the same as $\frac{4}{8}$ of an orange. In the whole orange we shall have 8 eighths, therefore 4 eighths must be the same as one-half.

Examples.

1. Reduce to twelfths the fractions $\frac{5}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$. *Ans.* $\frac{30}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$.

2. Reduce to 16ths, $\frac{3}{4}$, $\frac{3}{2}$, $\frac{3}{8}$. *Ans.* $\frac{12}{16}$, $\frac{24}{16}$, $\frac{6}{16}$.

3. Reduce to 20ths, $\frac{2}{5}$, $\frac{1}{4}$, $\frac{3}{2}$, $\frac{3}{10}$. *Ans.* $\frac{8}{20}$, $\frac{5}{20}$, $\frac{30}{20}$, $\frac{6}{20}$.

4. Reduce to 24ths, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{5}{12}$. *Ans.* $\frac{16}{24}$, $\frac{4}{24}$, $\frac{18}{24}$, $\frac{3}{24}$, $\frac{10}{24}$.

5. Reduce to the least common denominator the fractions, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{4}$. *Ans.* $\frac{6}{12}$, $\frac{8}{12}$, $\frac{3}{12}$. Here the least common multiple of 2, 3, and 4, is 12 (that is, the least number which is exactly divisible by 2, 3, and 4). All the fractions may therefore be brought to 12ths; thus $\frac{1}{2} \times \frac{6}{6} = \frac{6}{12}$, $\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$, and $\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$, where the multipliers 6, 4, and 3 are found by dividing 12 respectively by the denominators 2, 3, and 4.

6. Reduce to the least common denominator, $\frac{1}{4}$, $\frac{2}{9}$, and $\frac{5}{6}$. *Ans.* $\frac{9}{36}$, $\frac{8}{36}$, $\frac{30}{36}$.

7. Reduce to the least common denominator, $\frac{5}{12}$, $\frac{3}{8}$, $\frac{5}{6}$, and $\frac{3}{16}$.
Ans. $\frac{20}{48}$, $\frac{18}{48}$, $\frac{40}{48}$, $\frac{9}{48}$.

8. Reduce $\frac{12}{16}$ to its least terms. *Ans.* $\frac{3}{4}$. Here, dividing numerator and denominator by 4, we have, $\frac{12}{16} \div \frac{4}{4} = \frac{3}{4}$.

9. Reduce to their least terms, $\frac{8}{10}$, $\frac{6}{8}$, $\frac{16}{32}$, $\frac{4}{24}$, $\frac{18}{32}$, $\frac{15}{25}$, $\frac{8}{20}$, $\frac{16}{48}$, $\frac{35}{49}$, $\frac{72}{96}$, $\frac{18}{81}$, $\frac{21}{72}$. *Ans.* $\frac{4}{5}$, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{6}$, $\frac{9}{16}$, $\frac{3}{5}$, $\frac{2}{5}$, $\frac{1}{3}$, $\frac{5}{7}$, $\frac{3}{4}$, $\frac{2}{9}$, $\frac{7}{24}$.

47. *To add and subtract Fractions.*

It has already been shown, that when two fractions have the same denominator, then we can add them together or take their difference; because fractions in this form are expressed in the same fractional unit or piece of the whole unit. Hence we have the following rule: —

Rule. Reduce (if necessary) the fractions to the same denominator, add or subtract the numerators, as the case may be, and retain the common denominator.

1. $\frac{2}{3} + \frac{5}{3} = \frac{7}{3} = 2\frac{1}{3}$. See fig. to Art. 42. page 62.

2. $\frac{5}{3} - \frac{2}{3} = \frac{3}{3} = 1$.

3. Add $\frac{2}{5}$ and $\frac{1}{3}$. Bringing the fractions to 15ths, we have, $\frac{2}{5} + \frac{1}{3} = \frac{6}{15} + \frac{5}{15} = \frac{11}{15}$. *Ans.*

4. Add $2\frac{1}{4}$ and $3\frac{4}{5}$. Here we shall first add the fractional parts, and then add the whole numbers.

$\frac{1}{4} + \frac{4}{5} = \frac{5}{20} + \frac{16}{20} = \frac{21}{20} = 1\frac{1}{20}$; then $2 + 3 + 1\frac{1}{20} = 6\frac{1}{20}$. *Ans.*

5. Add $\frac{1}{5}$, $\frac{3}{8}$, and $\frac{7}{10}$. Here the least common multiple of 5, 8, and 10, is 40; we shall therefore bring the fractions to 40ths.

$$\frac{1}{5} + \frac{3}{8} + \frac{7}{10} = \frac{8}{40} + \frac{15}{40} + \frac{28}{40} = \frac{51}{40} = 1\frac{11}{40}.$$

Examples.

Find the value of

(1.) $\frac{1}{2} + \frac{5}{6}$	(2.) $\frac{3}{5} + \frac{3}{2}$	(3.) $\frac{5}{7} + \frac{2}{3}$
(4.) $1\frac{1}{2} + 2\frac{3}{4}$	(5.) $\frac{1}{2} + \frac{2}{3} + \frac{1}{5}$	(6.) $\frac{5}{6} - \frac{2}{3}$
(7.) $\frac{8}{9} - \frac{2}{3}$	(8.) $3\frac{1}{2} - 1\frac{1}{4}$	(9.) $\frac{1}{8} + \frac{3}{4} + \frac{5}{12}$
(10.) $\frac{2}{7} + \frac{4}{21} + \frac{1}{3}$	(11.) $\frac{2}{3} + \frac{5}{6} + \frac{7}{12}$	(12.) $19\frac{2}{3} + 1\frac{2}{7}$
(13.) $\frac{5}{7} + \frac{8}{9} + \frac{2}{3}$	(14.) $2\frac{1}{2} + 1\frac{2}{3} + \frac{1}{9}$	(15.) $\frac{2}{9} + \frac{3}{7} + \frac{4}{11}$
(16.) $\frac{2}{21} + \frac{9}{31}$	(17.) $\frac{3}{7} - \frac{2}{21}$	(18.) $3\frac{1}{2} - 2\frac{3}{8}$
(19.) $\frac{7}{15} - \frac{2}{21}$	(20.) $\frac{5}{19} - \frac{1}{38}$	(21.) $\frac{3}{7} + \frac{1}{21} + \frac{1}{3}$
(22.) $\frac{2}{5} - \frac{1}{4} + \frac{1}{2}$	(23.) $3\frac{2}{5} + \frac{4}{3} - \frac{1}{6}$	(24.) $3\frac{2}{5} - 1\frac{5}{8}$

Answers.

(1.) $1\frac{1}{3}$	(2.) $2\frac{1}{10}$	(3.) $1\frac{8}{21}$	(4.) $4\frac{1}{4}$	(5.) $1\frac{11}{30}$
(6.) $\frac{1}{6}$	(7.) $\frac{3}{8}$	(8.) $2\frac{1}{4}$	(9.) $1\frac{7}{24}$	(10.) $\frac{17}{21}$
(11.) $2\frac{1}{12}$	(12.) $20\frac{20}{21}$	(13.) $2\frac{17}{63}$	(14.) $4\frac{5}{18}$	(15.) $1\frac{10}{693}$
(16.) $\frac{251}{651}$	(17.) $\frac{1}{3}$	(18.) $1\frac{1}{8}$	(19.) $\frac{13}{36}$	(20.) $\frac{9}{35}$
(21.) $\frac{17}{21}$	(22.) $\frac{13}{20}$	(23.) $4\frac{7}{15}$	(24.) $1\frac{31}{40}$.	

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48. *To multiply Fractions, or to find the Fraction of a Fraction.*

1. To show that $\frac{3}{4}$ of $\frac{2}{3} = \frac{6}{12} = \frac{1}{2}$. *Where we multiply the numerators together for the new numerator and the denominators together for the new denominator:* thus $\frac{3}{4}$ of $\frac{2}{3} = \frac{3 \times 2}{4 \times 3} = \frac{6}{12}$.

Let *r e b c* be a unit (see fig. to Art. **46.** p. 65.). The space *r e b a* forms $\frac{2}{3}$. This is cut into 4 parts by the horizontal lines, and therefore the space *e d o b* will be $\frac{3}{4}$ of $\frac{2}{3}$, which we see contains 6 twelfths, that is, $\frac{3}{4}$ of $\frac{2}{3} = \frac{6}{12}$. The various steps in the operation may be written as follows :—

$$\tfrac{1}{4} \text{ of } \tfrac{1}{3} = \tfrac{1}{12};$$

but this result must be taken 2 times to give the fourth of 2 thirds.

$$\therefore \tfrac{1}{4} \text{ of } \tfrac{2}{3} = 2 \text{ times } \tfrac{1}{12} = \tfrac{2}{12}.$$

Now this result must be taken 3 times to give 3 fourths of two-thirds.

$$\therefore \tfrac{3}{4} \text{ of } \tfrac{2}{3} = 3 \text{ times } \tfrac{2}{12} = \tfrac{6}{12}.$$

2. $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$; because by $\frac{3}{4} \times \frac{2}{3}$ we mean $\frac{1}{3}$ of 2 times $\frac{3}{4} = \frac{6}{12}$. Hence it follows that *we multiply fractions in the same way that we find the value of a fraction of a fraction.*

3. $\frac{3}{4}$ of $\frac{8}{5} = 3$ times $\frac{2}{5} = \frac{6}{5} = 1\frac{1}{5}$. This example exhibits the principle of cancelling, where the 4 in the denominator is contained 2 times in the 8 of the numerator: thus we also have, $\frac{3}{10}$ of $\frac{15}{7} = \frac{3 \times 3}{2 \times 7} = \frac{9}{14}$.

4. When there are mixed numbers to multiply together, it is generally most convenient to reduce the mixed numbers to improper fractions before multiplying: thus, $2\frac{2}{3} \times 4\frac{3}{5} = \frac{8}{3} \times \frac{23}{5} = \frac{184}{15} = 12\frac{4}{15}$.

Examples.

(1.)	$\frac{1}{2}$ of $\frac{3}{4}$	(2.)	$\frac{5}{6}$ of $\frac{3}{7}$
(3.)	$\frac{2}{3}$ of $\frac{5}{4}$	(4.)	$\frac{2}{5}$ of $\frac{3}{4}$ of $\frac{6}{7}$
(5.)	$\frac{2}{7}$ of $\frac{3}{8}$	(6.)	$\frac{7}{8}$ of $\frac{4}{5}$ of $\frac{1}{2}$
(7.)	$\frac{1}{5}$ of $\frac{2}{9}$ of $\frac{3}{8}$	(8.)	$\frac{4}{5}$ of $\frac{7}{8}$ of $\frac{3}{7}$
(9.)	$\frac{3}{2} \times \frac{4}{9}$	(10.)	$\frac{3}{7} \times \frac{8}{9} \times \frac{14}{9}$
(11.)	$\frac{2}{11} \times \frac{4}{5} \times 5\frac{1}{2}$	(12.)	$\frac{5}{2} \times \frac{4}{10} \times 5$
(13.)	$2\frac{1}{2} \times 5\frac{1}{5}$	(14.)	$\frac{3}{8} \times 7 \times 2\frac{1}{2}$
(15.)	$\frac{1}{2}$ of $\frac{1}{4} + 3\frac{1}{2}$	(16.)	$\frac{3}{5}$ of $\frac{1}{6} + \frac{1}{5}$
(17.)	$\frac{2}{3}$ of $\frac{7}{8} + \frac{1}{2}$	(18.)	$\frac{1}{5}$ of $\frac{2}{3} + \frac{2}{3}$ of $\frac{9}{4}$
(19.)	$\frac{5}{6}$ of $\frac{9}{4} - \frac{3}{4}$ of $\frac{1}{9}$	(20.)	$\frac{7}{8}$ of $\frac{4}{5} + \frac{5}{6}$ of $\frac{3}{10}$
(21.)	$5\frac{1}{2} \times 2\frac{2}{3} + \frac{5}{9} \times \frac{3}{8}$	(22.)	$2\frac{5}{6} \times \frac{3}{2} - \frac{2}{3} \times \frac{9}{5}$

Answers.

(1.) $\frac{3}{8}$	(2.) $\frac{5}{14}$	(3.) $\frac{5}{6}$	(4.) $\frac{9}{35}$	(5.) $\frac{3}{28}$
(6.) $\frac{7}{20}$	(7.) $\frac{1}{60}$	(8.) $\frac{3}{10}$	(9.) $\frac{2}{3}$	(10.) $\frac{16}{27}$
(11.) $\frac{4}{5}$	(12.) 5	(13.) 13	(14.) $6\frac{9}{16}$	(15.) $3\frac{5}{8}$
(16.) $\frac{3}{10}$	(17.) $1\frac{1}{12}$	(18.) $1\frac{19}{30}$	(19.) $1\frac{19}{24}$	(20.) $\frac{19}{20}$
(21.) $14\frac{7}{8}$	(22.) $3\frac{1}{20}$.			

49. ***Examples in Compound Multiplication, when there is a Fraction in the Multiplier.***

	£	s.	d.		
1.	3	4	$6\frac{1}{4} \times 37\frac{2}{3}$		
			6		
	19	7	$1\frac{1}{2}$		
			6		
	116	2	9	product by	36
	3	4	$6\frac{1}{4}$	„	1
	2	3	$0\frac{1}{6}$	„	$\frac{2}{3}$
	121	10	$3\frac{5}{12}$	„	$37\frac{2}{3}$

In this example, the value of the $\frac{2}{3}$ is found by multiplying the price of one article by 2, and then dividing by 3, as explained in Art. 41.

2. $27\frac{3}{4}$ at £9 10*s.* 4*d.* *Ans.* £264 1*s.* 9*d.*
3. $34\frac{5}{8}$ at £8 7*s.* $2\frac{1}{4}$*d.* *Ans.* £291 3*s.* $8\frac{3}{8}$*d.*
4. $47\frac{7}{8}$ at £13 2*s.* $5\frac{3}{4}$*d.* *Ans.* £628 6*s.* $2\frac{9}{32}$*d.*
5 $77\frac{1}{3}$ at £15 19*s.* 6*d.* *Ans.* £1235 8*s.* 0*d.*
6. $37\frac{1}{8}$ at £2 14*s.* 9*d.* *Ans.* £103 13*s.* $7\frac{7}{8}$*d.*
7. $27\frac{5}{8}$ at £3 16*s.* 6*d.* *Ans.* £106 9*s.* 3*d.*
8. $38\frac{2}{3}$ at £7 19*s.* 9*d.* *Ans.* £308 17*s.* 0*d.*
9. $96\frac{3}{5}$ at £5 4*s.* 3*d.* *Ans.* £503 10*s.* $6\frac{3}{5}$*d.*
10. $55\frac{2}{3}$ at £11 9*s.* 6*d.* *Ans.* £638 15*s.* 6*d.*

50. *To divide Fractions.*

1. How often can $\frac{1}{2}$ be taken out of $\frac{3}{4}$? *Ans.* $1\frac{1}{2}$.

Here first bringing the fractions to the same denominator, $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \div \frac{2}{4} = 3 \div 2 = \frac{3}{2}$; because 2 fourths will be contained the same number of times in 3 fourths that 2 units are contained in 3 units.

2. Divide $\frac{3}{4}$ by $\frac{2}{5}$. Bringing the fractions to the same denominator, $\frac{3}{4} \div \frac{2}{5} = \frac{15}{20} \div \frac{8}{20}$; but $\frac{15}{20} \div \frac{1}{20} = 15$; $\therefore \frac{15}{20} \div \frac{8}{20} =$ the eighth of $15 = \frac{15}{8}$.

Here we observe that this result is obtained by *inverting the devisor, and then multiplying;* thus, $\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$.

Examples.

(1.) $\frac{1}{2} \div \frac{1}{4}$ (2.) $\frac{3}{2} \div \frac{3}{8}$ (3.) $2\frac{1}{2} \div \frac{3}{4}$
(4.) $\frac{2}{3} \div \frac{4}{5}$ (5.) $\frac{7}{8} \div 2\frac{1}{4}$ (6.) $3\frac{1}{4} \div 2\frac{5}{6}$
(7.) $\frac{4}{5} \div 7\frac{3}{10}$ (8.) $3\frac{9}{10} \div 2\frac{3}{5}$ (9.) $\frac{1}{2}$ of $\frac{3}{4} \div \frac{7}{8}$
(10.) $\frac{3}{4}$ of $\frac{2}{9} \div \frac{2}{7}$ of $\frac{5}{8}$ (11.) $\frac{2}{5}$ of $3\frac{1}{2} \div \frac{3}{7}$ of $2\frac{1}{4}$.

Answers.

(1.) 2 (2.) 4 (3.) $3\frac{1}{3}$ (4.) $\frac{5}{6}$ (5.) $\frac{7}{18}$ (6.) $2\frac{5}{17}$
(7.) $\frac{8}{73}$ (8.) $1\frac{1}{2}$ (9.) $\frac{3}{7}$ (10.) $\frac{14}{15}$ (11.) $1\frac{61}{135}$.

51. In practice it is often convenient to proceed after the following method.

1. If $39\frac{3}{4}$ articles cost £84 5*s.* $2\frac{1}{4}$*d.*, what is the cost of 1 article?

	£	s.	d.	
$39\frac{3}{4}$)	84	5	$2\frac{1}{4}$ (	
4			4	
159)	337	0	9	(£2 2s. $4\frac{117}{159}$d.
	318			
	19			
	&c.			

Here to get rid of the fraction, we multiply the divisor by 4, and at the same time the dividend by 4, and then proceed with the division. If we take the number of articles 4 times, it is obvious that we must increase the price 4 times.

	£	s.	d.			£	s.	d.	
(1.)	34	17	4	$\div 5\frac{1}{4}$	(2.)	25	4	$8\frac{3}{4}$	$\div 2\frac{1}{3}$
(3.)	7	4	8	$\div 9\frac{2}{5}$	(4.)	15	5	7	$\div 18\frac{1}{2}$
(5.)	14	17	$2\frac{1}{2}$	$\div 12\frac{2}{3}$	(6.)	5	2	3	$\div 22\frac{1}{7}$

Answers.

(1.)	6	12	$9\frac{3}{4}+$	(2.)	10	16	$3\frac{3}{4}$	(3.)	0	15	$4\frac{1}{2}+$
(4.)	0	16	$6\frac{8}{37}$	(5.)	1	3	$5\frac{1}{2}+$	(6.)	0	4	$7\frac{1}{4}+$

52. *To change the Denomination of a Quantity.*

1. Reduce 7s. 4d. to the fraction of a shilling.
Here 7s. 4d. = 88d. = $\frac{88}{12}$s. = $\frac{22}{3}$s.

2. Reduce 3 farthings to the fraction of a pound.

$$\tfrac{3}{4}d. = £\frac{3}{4\times 12\times 20} = £\frac{1}{320} \quad \textit{Ans.}$$

(1.) Reduce to the fraction of a shilling, $2\frac{1}{2}$d., $\frac{1}{2}$d., $\frac{2}{3}$d., $6\frac{3}{4}$d., $9\frac{1}{2}$d., 1s. $7\frac{1}{2}$d. *Ans.* $\frac{5}{24}$s., $\frac{1}{24}$s., $\frac{1}{18}$s., $\frac{9}{16}$s., $\frac{19}{24}$s., $\frac{13}{8}$s.

(2.) Reduce to the fraction of a pound, 2s. 4d., 5s. 2d., 2s. $4\frac{1}{4}$d., £5 2s. 6d., £2 4s., £6 0s. 2d.
Ans. £$\frac{7}{60}$, £$\frac{31}{120}$, £$\frac{113}{960}$, £$\frac{41}{8}$, £$\frac{11}{5}$, £$\frac{721}{120}$.

(3.) Reduce £$2\frac{1}{7}$ to the fraction of a penny. *Ans.* $\frac{3600}{7}$d.

(4.) Reduce $\frac{1}{12}$ lb. Av. to the fraction of an oz.
Ans. $1\frac{1}{3}$ oz.

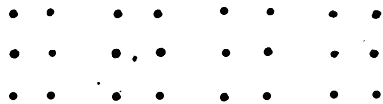
(5.) Reduce $2\frac{1}{2}$ ft. to the fraction of a yard. *Ans.* $\frac{5}{6}$ yds.

53. *Useful Properties of Numbers.*

1. Numbers may be multiplied and divided in any order; thus to calculate $\frac{8 \times 3}{4}$, we have,

$$\text{First}\qquad \begin{array}{r} 8 \\ 3 \\ \hline 4\,)\;24 \\ \hline 6 \end{array} \qquad\qquad \text{Second}\qquad \begin{array}{r} 4\,)\;8 \\ \hline 2 \\ 3 \\ \hline 6 \end{array}$$

The annexed arrangement of dots or counters shows that these operations must produce the same result; 1st, the number in one row taken 3 times gives the total number of counters, and the 4th of this number is the counters in one group: 2nd, the 4th of the counters in one row is the number in one row of each group, and this number taken 3 times is the counters in one group, as before.



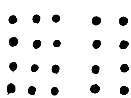
In like manner when the result is a fraction, we have 5 times $(8 \div 3) = 5$ times $\frac{8}{3} = \frac{40}{3}$.

2. When the product of two quantities is to be divided by any number, we divide *one* of the factors; thus $(3 \times 8) \div 4 = 3 \times 2$. The preceding arrangement of counters establishes this property.

Whereas, it is shown, in Art. 24. that in the case $(12 + 8) \div 4$, we divide *all* the numbers within the vinculum by the divisor.

3. When the product of two quantities is to be multiplied by any number, we multiply this number into *one* of the factors; thus $(3 \times 2) \times 4 = 3 \times 8$. This property is proved by the preceding arrangement of counters.

Whereas, in the case, $(3 + 2) \times 4$, we multiply all the numbers within the vinculum by the multiplier; thus $(3+2) \times 4 = 3 \times 4 + 2 \times 4$. Proof



that is, $(3 + 2) \times 4 = 3 \times 4 + 2 \times 4$.

4. If the same number be added to, or subtracted from equal quantities, the results will still be equal.

Ex. 1. If 2 be taken from an unknown number, the remainder will be 3; required the number?

Here, number $-2 = 3$; adding 2 to these equals,

number $= 5$.

Ex. 2. If 4 be added to a required number, it will produce 6.

Here, number $+ 4 = 6$; subtracting 4 from these equals,

number $= 2$.

5. If equal quantities be multiplied or divided by the same number, the results will still be equal.

Ex. 1. One-third of my money is 4*s*., how much have I?

Here, $\frac{1}{3}$ of money $=$ 4*s*.; taking these equals 3 times,

money $=$ 12*s*.

Ex. 2. I have $\frac{2}{9}$ of a property, and my share is valued at £8, what is the worth of the whole?

Here, $\frac{2}{9}$ of property $=$ £8,

first dividing these equals by 2, and then multiplying by 9, we have,

$$\text{property} = \frac{£8}{2} \times 9 = £36.$$

For further applications of these axioms, see Rule of Three.

DECIMAL FRACTIONS.

54. If we carry the decimal scale of notation (explained in Art. 7.) below unity, and indicate by a point where the fractional parts begin, we shall have for example, $53{\cdot}24 = 5$ tens $+ 3$ units $+ 2$ tenths $+ 4$ hundredths $= 50 + 3 + \frac{2}{10} + \frac{4}{100}$. The figures after the decimal point are called decimal fractions, or simply *decimals.* This extension of the common numeration scale, it will be observed, is only another way of writing down fractions of tenths, hundredths, thousandths, &c.

55. *To change a Decimal into a Fraction, and conversely.*

1. Put 4·35 into the form of a fraction.

$4·35 = 4 + \frac{3}{10} + \frac{5}{100} =$ (bringing the fractions to the same denominator, and adding) $\frac{400}{100} + \frac{30}{100} + \frac{5}{100} = \frac{435}{100}$. Where we put as many ciphers in the denominator as there are decimal figures.

2. Put $\frac{562}{100}$ into the form of a decimal.

$\frac{562}{100} = \frac{500}{100} + \frac{60}{100} + \frac{2}{100} = 5 + \frac{6}{10} + \frac{2}{100} = 5·62$. Where there are as many decimal figures as there are ciphers in the denominator.

Observation. Adding ciphers to the right of a decimal does not alter its value; thus, ·2 is the same as ·200, for each of them is $\frac{2}{10}$.

Examples.

1. Put into the form of a fraction,

·27, 3·5, 1·02, ·02, ·0031, 52·032, ·0001.

Ans. $\frac{27}{100}$, $\frac{35}{10}$, $\frac{102}{100}$, $\frac{2}{100}$, $\frac{31}{10000}$, $\frac{52032}{1000}$, $\frac{1}{10000}$.

2. Put into the form of a fraction,

$\frac{3}{10}$, $\frac{432}{100}$, $\frac{423}{1000}$, 73 tenths, 425 hundredths, 7 units + 4 hundredths, 4 hundreds + 3 units + 5 hundredths, $\frac{6245}{100}$, $\frac{25}{10000}$, $\frac{3}{100000}$.

Ans. ·3, 4·32, ·423, 7·3, 4·25, 7·04, 403·05, 62·45, ·0025, ·00003.

56. *To add and subtract Decimal Fractions.*

Ex. 2·43 5·32 ·58 8·33	Here the addition of the hundredths gives $\frac{13}{100}$, which equal $\frac{1}{10}$ and $\frac{3}{100}$; we therefore put 3 in the hundredths' place and carry the 1-tenth to the column of tenths, and so on as in common addition.

Ex. 5·35 2·48 2·87	Here we cannot take 8 hundredths from 5 hundredths, but we borrow $\frac{1}{10}$, or $\frac{10}{100}$, from the $\frac{3}{10}$, and then 8 hundredths from 15 hundredths and 7 hundredths remain; we

have now to take 4 tenths from 2 tenths, or, as in common subtraction, 5 tenths from 3 tenths, and so on.

Examples.

Find the values of

1. $23{\cdot}76+7{\cdot}92+12{\cdot}87$. *Ans.* $44{\cdot}55$.
2. $3{\cdot}7+{\cdot}4+{\cdot}02+1{\cdot}39$. *Ans.* $5{\cdot}51$.
3. $3{\cdot}045+{\cdot}02+32{\cdot}48+{\cdot}002$. *Ans.* $35{\cdot}547$.
4. ${\cdot}007+3{\cdot}02+{\cdot}5+1{\cdot}234$. *Ans.* $4{\cdot}761$.
5. $32+{\cdot}4+1{\cdot}02+{\cdot}891$. *Ans.* $34{\cdot}311$.
6. $4{\cdot}3-2{\cdot}5$, and $4{\cdot}58-3{\cdot}72$. *Ans.* $1{\cdot}8$, *and* ${\cdot}86$.
7. $31{\cdot}21-13{\cdot}04$, and $4-3{\cdot}71$. *Ans.* $18{\cdot}17$, *and* ${\cdot}29$.
8. $2{\cdot}457-1{\cdot}68$, and $14{\cdot}5-7{\cdot}068$. *Ans.* ${\cdot}777$, *and* $7{\cdot}432$.
9. $5{\cdot}6-{\cdot}002$, and ${\cdot}3-{\cdot}0275$. *Ans.* $5{\cdot}598$, *and* ${\cdot}2725$.

57. *To multiply and divide Decimal Fractions.*

Ex.
$$\begin{array}{r} 1{\cdot}23 \\ \underline{2{\cdot}5} \\ 615 \\ \underline{246} \\ 3{\cdot}075 \end{array}$$

Here we multiply as in ordinary multiplication, and mark off in the product as many decimal figures as there are in the multiplier and multiplicand together;

because $1{\cdot}23\times2{\cdot}5=\frac{123}{100}\times\frac{25}{10}=\frac{3075}{1000}=3{\cdot}075$.

Ex. $2{\cdot}5)3{\cdot}075(1{\cdot}23$. Here we divide as in ordinary division, and then mark off, in the quotient, as many decimal places as are equal to the difference of the number in the dividend and divisor; the reason of this process is manifest, as it is just the reverse of the rule for multiplication.

Examples.

(1.) $3{\cdot}24\times2{\cdot}5$	(2.) $12{\cdot}3876\times5$
(3.) $256{\cdot}9743\times{\cdot}25$	(4.) ${\cdot}023\times{\cdot}34$
(5.) ${\cdot}001346532\times{\cdot}027$	(6.) ${\cdot}0363\times51{\cdot}2$
(7.) ${\cdot}01\times{\cdot}001$	(8.) ${\cdot}002\times500$
(9.) $4{\cdot}52\times55{\cdot}3$	(10.) $8{\cdot}002\times{\cdot}004$
(11.) $6{\cdot}32\div2{\cdot}4$	(12.) ${\cdot}34\div{\cdot}21$
(13.) $345{\cdot}2\div4{\cdot}73$	(14.) $236{\cdot}4\div{\cdot}0021$
(15.) $18\div5{\cdot}34$	(16.) $14{\cdot}2\div45{\cdot}02$
(17.) $1\div{\cdot}01$	(18.) ${\cdot}002\div34{\cdot}2$.

Answers.

(1.) 8·1	(2.) 61·938	(3.) 64·23575
(4.) ·00782	(5.) ·000036356364	(6.) 1·85856
(7.) ·00001	(8.) 1	(9.) 249·956
(10.) ·032008	(11.) 2·633′	(12.) 1·619+
(13.) 72·9809	(14.) 112571·42+	(15.) 3·3707
(16.) ·3154	(17.) 100	(18.) ·0000584.

58. *To convert any Fraction into a Decimal.*

As a fraction is equal to the numerator divided by the denominator (Art. **41.**) we may readily express any fraction in form of a decimal fraction, by dividing the numerator by the denominator.

Ex. Reduce $\frac{11}{4}$ to the form of a decimal fraction.

$$\begin{array}{r} 4)\underline{11{\cdot}00} \\ 2{\cdot}75 \end{array}$$

Here the 4th of 11 units is 2 units, with 3 units over, which are equal to 30 tenths; then the 4th of 30 tenths is 7 tenths, with 2 tenths over, which are equal to 20 hundredths, and so on, the operation being, in effect, the same as in common division.

Examples.

1. Reduce to decimals, $\frac{2}{5}$, $\frac{8}{5}$, $\frac{5}{8}$, $\frac{3}{7}$, $\frac{4}{13}$, $\frac{2}{23}$, $\frac{3}{16}$, $\frac{2}{3}$.

Answers. ·4, 1·6, ·625, ·4285+, ·30769, ·0869, ·1875, ·666′.

59. *To find the Value of a Decimal of a given Quantity.*

Ex. 1. What is the value of £·375?

$$\begin{array}{r} \cdot375 \\ \underline{20} \\ 7\cdot500 \\ \underline{12} \\ 6\cdot000 \end{array}$$

Here we multiply by 20 to bring the pounds to shillings, and then the decimal part of the shillings by 12 to bring them to pence.

Ans. 7*s.* 6*d.*

Examples.

Find the value of

1. ·375 of a shilling; ·75 lbs. Av.; £·0645; £·156; £·007; ·12 gui.; ·32 acres; ·63 sq. yds.; ·57 acres; ·45 acres; £3·75.

Answers. $4\frac{1}{2}$; 12 oz.; 1*s.* $3\frac{1}{4}$*d.*; 3*s.* $1\frac{1}{4}$*d.*; $1\frac{1}{2}$*d.* +; 2*s.* 6*d.* +; 1 r. 11 p. 6 yds. +; 5 sq. ft. 96 in. +; 2 r. 11 p. 6 yds. +; 1 r. 32 p.; £3 15*s.*

60. ***To reduce a Quantity to the Decimal of another.***

1. Reduce $9\frac{1}{2}$*d.* to the decimal of a shilling.

Here $9\frac{1}{2}d. = \frac{19}{2}d. = \frac{19}{2 \, . \, 12} s. = \cdot 79166' s.$

2. Reduce £2 7*s.* $4\frac{3}{4}$*d.* to the decimal of a pound.

$$
\begin{array}{r}
4)3\cdot 00 \\
\hline
12)4\cdot 75d. \\
\hline
20)7\cdot 3958s. \\
\hline
2\cdot 36979 + £ \ \textit{Ans.}
\end{array}
$$

Here we bring the 3 farthings to the decimal of a penny by dividing by 4, which gives ·75*d.*, then prefixing the 4*d.* we have 4·75*d.* to bring to shillings, which we do by dividing by 12, and so on.

Examples.

(1.) Reduce to the decimal of a pound, 5*s.*; 4*s.*; 5*s.* 6*d.*; 2*s.* 6*d.*; 3*s.* 4*d.*; 7*s.* 4*d.*; 9*s.* $5\frac{1}{2}$*d.*; £2 3*s.* 8*d.*; £5 6*s.* $2\frac{1}{2}$*d.*; 18*s.* 6*d.*; £8 14*s.* $9\frac{1}{4}$*d.*

Ans. £·25; £·2; £·275; £·125; £·1666′; £·366′; £·4729; £2·183′; £5·3104; £·925; £8·7385.

(2.) Reduce to the decimal of a penny, $3\frac{1}{2}$*d.*; $4\frac{3}{4}$*d.*; $\frac{1}{4}$*d.*; $6\frac{1}{4}$*d.*; $2\frac{3}{5}$*d.* *Ans.* 3·5*d.*; 4·75*d.*; ·25*d.*; 6·25*d.*; 2·6*d.*

(3.) Reduce to the decimal of a shilling, $6\frac{1}{2}$*d.*; $4\frac{3}{4}$*d.*; $1\frac{1}{4}$*d.*; 2*s.* $4\frac{1}{2}$*d.*; 5*s.* $7\frac{3}{4}$*d.*; 3*s.* $8\frac{1}{2}$*d.*

Ans. ·5416 + *s.*; ·3958 + *s.*; ·10416*s.*; 2·375*s.*; 5·6458*s.*; 3·7083′*s.*

(4.) Reduce to the decimal of a lb. Av., $3\frac{1}{2}$ oz.; $5\frac{1}{4}$ oz.; 7 lbs. $2\frac{3}{4}$ oz.; 3 lbs. $6\frac{2}{5}$ oz.

Ans. ·21875 lbs.; ·328125 lbs.; 7·171875 lbs.; 3·4 lbs.

(5.) Reduce to the decimal of a yard, 3 ft. 6 in.; 2 ft. 9 in.; 1 ft. 7 in.; 1 yd. 2 ft. 10 in.

Ans. 1·166′; ·9166′; ·5277′; 1·94′.

61. PRACTICE.

Ex. What is the amount of 27 articles at £2 14*s.* 6*d.*?

	27		
	2		
	54		
10*s.* = $\frac{1}{2}$	13	10	
4*s.* = $\frac{1}{5}$	5	8	
6*d.* = $\frac{1}{8}$		13	6
	£73	11	6

Here 27 at £2 will amount to £54, but as 27 at £1 amounts to £27, so therefore 27 at 10*s.* will be the $\frac{1}{2}$ of £27, and similarly 27 at 4*s.* will be the $\frac{1}{5}$ of £27, and lastly as 6*d.* is the $\frac{1}{8}$ of 4*s.*, the amount of 27 at 6*d.* will be $\frac{1}{8}$ of the value, corresponding to the 4*s.*; hence the addition of all these parts will give the amount of the whole.

Many of the following questions may be calculated by Multiplication as well as by Practice.

			£	*s.*	*d.*		£	*s.*	*d.*
(1.)	95	at	5	10	10	*Ans.*	526	9	2
(2.)	109	at	7	2	$3\frac{1}{2}$	„	775	9	$9\frac{1}{2}$
(3.)	145	at	13	3	6	„	1910	7	6
(4.)	240	at	6	17	$4\frac{1}{4}$	„	1648	5	0
(5.)	214	at	6	5	9	„	1345	10	6
(6.)	345	at	7	12	6	„	2630	12	6
(7.)	563	at	25	16	4	„	14534	15	8
(8.)	396	at	6	17	$7\frac{1}{4}$	„	2724	11	3
(9.)	$37\frac{2}{3}$	at	4	5	8	„	161	6	$9\frac{1}{3}$
(10.)	$640\frac{1}{4}$	at	24	8	6	„	15638	2	$1\frac{1}{2}$
(11.)	$324\frac{3}{8}$	at	7	9	8	„	2427	11	$10\frac{1}{4}$ +
(12.)	$784\frac{1}{2}$	at	5	6	4	„	4170	18	6

Ex. What is the amount of 348 articles at 7*s.* $4\frac{1}{2}$*d.*

	348	
	7	
	2436	
4*d.* = $\frac{1}{3}$	116	
$\frac{1}{2}$ = $\frac{1}{8}$	14	6
2,0	256,6	6
	£ 128 6	6

Here 348 at 7*s.* each will come to 2436*s.*, but as 348 at 1*s.* amounts to 348*s.*, so therefore 348 at 4*d.* will be the $\frac{1}{3}$ of 348*s.* or 116*s.*; and as $\frac{1}{2}$*d.* is the $\frac{1}{8}$ of 4*d.*, the amount of 348 at $\frac{1}{2}$*d.* will be the $\frac{1}{8}$ of the value corresponding to the 4*d.*; and so on.

		s.	d.		£	s.	d.
(1.)	235 at	6	2	*Ans.*	72	9	2
(2.)	176 at	7	6	„	66	0	0
(3.)	478 at	15	6	„	370	9	0
(4.)	581 at	17	4	„	503	10	8
(5.)	389 at	18	4	„	356	11	8
(6.)	863 at	12	4	„	532	3	8
(7.)	524 at	9	10	„	257	12	8
(8.)	728 at	13	9	„	500	10	0
(9.)	612 at	19	$6\frac{1}{2}$	„	597	19	6
(10.)	502 at	6	$4\frac{1}{4}$	„	159	9	$9\frac{1}{2}$
(11.)	324 at	7	$2\frac{3}{4}$	„	117	2	3

62. In the following example, all the terms of the given quantity are not expressed in the unit whose value is given.

Ex. 17 yds. 3 qrs. at 8*s.* 4*d.* per yard.

	17			
	8			
	136			cost 17 at 8*s.*
4*d.* = $\frac{1}{3}$	5	8		„ „ at 4*d.*
	6	3		„ 3 qrs.
20	147	11		
	£7	7	11	

	s.	d.	
	8	4	
2 = $\frac{1}{2}$	4	2	cost 2 qrs.
1 = $\frac{1}{2}$	2	1	„ 1 qr.
	6	3	„ 3 qrs.

Or thus,

		s.	d.	
		8	4	
			17	
	7	1	8	cost 17 yds.
2 qrs. = $\frac{1}{2}$		4	2	cost 2 qrs.
1 qr. = $\frac{1}{2}$		2	1	cost 1 qr.
	£7	7	11	cost 17 yds. 3 qrs.

Examples.

1. 69 lbs. 3 oz. at 9*s.* 4*d.* per lb.? *Ans.* £32 5*s.* 9*d.*
2. 157 cwt. 1 qr. 14 lbs. at £6 10*s.* 8*d.* per cwt.? *Ans.* £1028 3*s.* 8*d.*
3. 63 lbs. 12 oz. of tea, at 5*s.* $8\frac{1}{2}$*d.* per lb.? *Ans.* £18 3*s.* $10\frac{7}{8}$*d.*
4. 114 cwt. 2 qr. 6 lbs. at £5 8*s.* 6*d.* per cwt.? *Ans.* £621 9*s.* $0\frac{3}{4}$*d.*

5. 218 cwt. 3 qr. 8 lbs. at £7 5*s.* 4*d.* per cwt.?
Ans. £1590 2*s.* $0\frac{4}{7}$*d.*

6. 314 yds. 2 qr. 3 nls. at 6*s.* 5*d.* per yard?
Ans. £100 19*s.* $2\frac{15}{16}$*d.*

7. 213 yds. 1 qr. 2 nls. at 8*s.* $2\frac{1}{4}$*d.* per yard?
Ans. £87 7*s.* $0\frac{3}{32}$*d.*

8. 114 yds. 2 ft. 3 in. at 4*s.* 8*d.* per yard? *Ans.* £26 15*s.* 6*d.*

9. 671 sq. yds. 4 sq. ft. at 6*s.* 9*d.* per sq. yard?
Ans. £226 12*s.* 3*d.*

10. 18 ac. 3 r. 10 p. at £2 15*s.* per acre? *Ans.* £51 14*s.* $8\frac{1}{4}$*d.*

11. Required a labourer's wages for 5 w. 4 da. at 18*s.* 6*d.* per week? *Ans.* £5 4*s.* 10*d.*

RULE OF THREE.

63. This rule includes all those methods whereby a fourth term is found from three given terms in a problem.

Several methods have already been given for solving questions in this rule. (See Art. **33.**) The subject will now be treated in a more general manner, that is, by methods which are not restricted to the peculiar data of the problem.

64. *By Multiplication and Division.*

1. If 7 sheep cost £6, what will 20 sheep cost at the same rate?

Cost 7 sheep = £6.

$\therefore$ Cost 1 sheep = $\frac{1}{7}$ of £6 = £$\frac{6}{7}$.

$\therefore$ Cost 20 sheep = 20 times £$\frac{6}{7}$ = $\frac{£6 \times 20}{7}$ = £17 2*s.* $10\frac{2}{7}$*d.*

Here, according to Property 1, Art. **53.**, we first multiply 6 and 20 together, and then divide by 7.

Or thus,

Cost 7 sheep = £6;

taking the number 20 times, and also the cost 20 times (see Property 5, Art. **53.**), we have,

Cost 140 sheep = 20 times £6 = £6 × 20;

taking the 7th part of the number, we have,

Cost 20 sheep $= \frac{1}{7}$ of £6 × 20 $= \frac{£6 \times 20}{7}$, as before.

2. If 6 chests of oranges cost £27 13*s.* 6*d.*, what will 33 cost? *Ans.* £152 4*s.* 3*d.*

3. How many yards of land can I purchase for £230, if 37 yards cost £9?

No. yds. for £9 = 37.

∴ No. yds. (or it may be parts of yds.) for £1 $= \frac{37}{9}$.

∴ No. yds. for £230 = 230 times $\frac{37}{9} = 945\frac{5}{9}$ yds.

Here we first multiply 230 by 37, and then divide by 9.

4. If 15 yds. can be purchased for 5*s.*, how many yards may be had for 11*s.*? *Ans.* 33 *yards.*

5. If 3 yards cost 12*s.*, how many can I purchase for 5*s.* 6*d.*? *Ans.* $1\frac{3}{8}$ *yards.*

6. If 3 cwt. 2 qr. 17 lbs. cost £13 16*s.* $7\frac{1}{4}d.$, what is that per cwt.? *Ans.* £3 15*s.* $8\frac{3}{4}d.$ +

Here we first reduce the weights to the same denomination, that is lbs., thus 3 cwt. 2 qr. 17 lbs. = 409 lbs.; and 1 cwt. = 112 lbs.

$$\text{Cost } 409 \text{ lbs.} = £13\ 16s.\ 7\tfrac{1}{4}d.$$

$$\therefore \text{Cost } 1 \text{ lb.} = \frac{£13\ 16s.\ 7\frac{1}{4}d.}{409}$$

$$\therefore \text{Cost } 112 \text{ lbs.} = \frac{£13\ 16s.\ 7\frac{1}{4}d. \times 112}{409} =$$

7. If I pay £42 for 9 cwts. 3 qrs. 7 lbs. of sugar, what would be the cost of 1 cwt. 2 qrs.? *Ans.* £6 8*s.* $4\frac{3}{4}d.$ +

8. What will be the cost of $34\frac{1}{2}$ yards, when 11 yards cost £3 19*s.* $0\frac{3}{4}d.$? *Ans.* £12 7*s.* $11\frac{5}{8}d.$

9. If the rate levied upon £37 be £9 8*s.* 4*d.*, how much is that in the pound? *Ans.* 5*s.* $1\frac{3}{37}d.$

10. What is the coach fare for $65\frac{1}{2}$ miles, if it is 14*s.* 8*d.* for 42 miles?

$$\text{Fare for 1 mile} = \frac{14s.\ 8d.}{42}$$

$$\text{Fare for } 65\tfrac{1}{2} \text{ miles} = \frac{14s.\ 8d. \times 65\frac{1}{2}}{42} = £1\ 2s.\ 10\tfrac{10}{21}d.$$

11. If a person walk 9 miles in 3 hours, in what time will he walk 41 miles?

miles walked in 1 hour $= \frac{9}{3} = 3$.

$\therefore$ No. hours in walking 41 miles $= \frac{41}{3} = 13\frac{2}{3}$ h.

Or thus,

No. hours (or parts of hours) in walking 1 mile $= \frac{3}{9}$ h.

" " " 41 miles = 41 times $\frac{3}{9}$ h. =

12. If a railway train move 87 miles in 2 hours, how many feet would it move in 1 minute? *Ans.* 3828 *feet.*

13. If a stick 3 feet long cast a shadow of 5 feet, what shadow will a stick 7 feet long cast? *Ans.* $11\frac{2}{3}$ *feet.*

Shadow of a stick 1 ft. long =

$\therefore$ " " 7 ft. " =

14. If 448 lbs. of salt cost 9*s.* 8*d.*, what weight of salt may be purchased for 14*s.* 5*d.*?

Here first reducing the money to the same name, 9*s.* 8*d.* = 116*d.*; and 14*s.* 5*d.* = 173*d.*

No. lbs. for 116*d.* = 448

$\therefore$ " " 1*d.* $= \frac{448}{116}$

$\therefore$ " " 173*d.* = 173 times $\frac{448}{116}$ lbs. $= 668\frac{4}{29}$.

Or thus,

Since 116*d.* buys 448 lbs., as many times as 116*d.* can be taken out of 173*d.*, so many times 448 lbs. will be purchased.

$\therefore$ No. lbs. for 173*d.* $= \frac{173}{116}$ times 448 lbs. =

15. If 5 yards of paper cost 4*s.* 7*d.*, how many yards may be purchased for £2 5*s.* 4*d.*? *Ans.* $49\frac{5}{11}$ *yds.*

16. How much cloth may be bought for 16*s.* 8*d.*, when 18 yds. cost £3 5*s.* 2*d.*? *Ans.* 4 *yds.* 2 *qr.* $1\frac{257}{391}$ *nls.*

17. If $2\frac{5}{6}$ cost 9*s.* 8*d.*, what will 1 cost?

Cost $2\frac{5}{6}$ = 9*s.* 8*d.*

taking the number 6 times to get rid of the fraction,

Cost 17 = 6 times 9*s.* 8*d.* = £2 18*s.*

$\therefore$ Cost 1 = £2 18*s.* ÷ 17 = 3*s.* $4\frac{16}{17}$*d.*

18. If $3\frac{1}{3}$ cost £4 10*s.*, what will 1 cost? *Ans.* £1 7*s.*

19. If $2\frac{1}{5}$ cost £3 6*s.*, what will 1 cost? *Ans.* £1 10*s.*

20. If $2\frac{1}{3}$ cost £1 15*s.*, what will $\frac{1}{4}$ cost? *Ans.* 3*s.* 9*d.*

21. If $1\frac{1}{7}$ cost £3 4*s.*, what will $\frac{1}{5}$ cost? *Ans.* 11*s.* $2\frac{2}{5}$*d.*

22. $\frac{3}{5}$ of a ship is worth £210, what is the value of the whole ship? *Ans.* £350.

23. If $\frac{2}{3}$ of an article cost 4*s.*, what will $\frac{3}{4}$ cost?

Here first reducing the fractions to the same denominator, $\frac{2}{3} = \frac{8}{12}$, and $\frac{3}{4} = \frac{9}{12}$, then,

Cost $\frac{8}{12} = 4s.$

$\therefore$ Cost $\frac{1}{12} = \frac{1}{8}$ of $4s. = \frac{4}{8}s.$

$\therefore$ Cost $\frac{9}{12} = 9$ times $\frac{4}{8}s. = 4s.\ 6d.$

24. If $2\frac{1}{2}$ cost 3*s.* 6*d.*, what will $1\frac{1}{8}$ cost?

Ans. 1*s.* $6\frac{3}{4}$*d.* +

25. If $3\frac{7}{9}$ cost 8*s.* 8*d.*, what will $2\frac{1}{3}$ cost? *Ans.* 5*s.* $4\frac{4}{17}$*d.*

26. If $7\frac{1}{3}$ cost 9*s.* 8*d.*, what will $2\frac{1}{2}$ cost?

Ans. 3*s.* $3\frac{1}{2}$*d.* +

27. If $2\frac{1}{4}$ cost 8*s.* 4*d.*, what will $3\frac{1}{8}$ cost?

Ans. 11*s.* $6\frac{3}{4}$*d.* +

28. $\frac{1}{7}$ of a property is worth £210, what is the value of $\frac{2}{5}$? *Ans.* £882.

29. How many yards of calico can I purchase for 18*s.* 6*d.* at the rate of $4\frac{1}{2}$*d.* a yard? *Ans.* $49\frac{1}{3}$ *yards.*

30. If $2\frac{2}{3}$ yards of cloth cost 12*s.* 6*d.*, how much can be bought for 16*s.* 8*d.*? *Ans.* 3 *yds.* $2\frac{2}{9}$ *qr.*

31. If the carriage of $6\frac{3}{4}$ tons be £5 8*s.* 9*d.*, what will be the carriage of $3\frac{2}{5}$ tons for the same distance?

Ans. £2 14*s.* $9\frac{1}{4}$*d.*

32. If 20 workmen can do a certain piece of work in 8 days, how long will it take 12 men to do the same?

No. days in which 1 man will do the work $= 20 \times 8 = 160$

$\therefore$ „ 12 men „ $= \frac{1}{12}$ of $160 = 13\frac{1}{3}$ days.

This is a question in what is called inverse proportion.

33. If 84 yds. of carpet 3 qrs. wide will cover a floor, how much of 5 qrs. wide will be required?

No. yds. 3 qrs. wide $= 84$.

$\therefore$ „ 1 qr. „ $= 3 \times 84 = 252$.

$\therefore$ „ 5 qrs. „ $= \frac{252}{5} = 50\frac{2}{5}$ yds.

34. If 5 men can do a certain piece of work in 9 days, how many men will do the same in 7 days? *Ans.* $6\frac{3}{7}$ *days.*

No. men to do it in 1 day =

$\therefore$ „ „ 7 days =

35. If 4 compositors can set up a work in 9 days, in what time will 3 perform the same? *Ans.* 12 *days.*

36. How many feet of matting 2 feet wide will cover a floor 30 feet long and 24 feet broad?

No. ft. 1 ft. wide = 24 times 30 = 720.

$\therefore$ „ 2 ft. „ $= \frac{1}{2}$ of 720 = 360 ft.

37. How many yards of cloth at 5*s.* must be given for 24 yards at 11*s.*?

Cost of the cloth = 11*s.* $\times$ 24 = 264*s.*

But every 5*s.* of this money purchases 1 yard,

$\therefore$ No. yards $= \frac{264}{5} = 52\frac{4}{5}$ yds.

38. How much land at £1 17*s.* an acre should be given in exchange for 340 acres at £2 5*s.* an acre? *Ans.* $413\frac{19}{37}$ *ac.*

39. If 52 tenpenny loaves are made out of a quarter of flour, what will be the price of the loaf when 60 are made out of the quarter? *Ans.* $8\frac{2}{3}$*d.*

40. What is the yearly income of a person who pays £12 10*s.* 6*d.* income tax at the rate of 7*d.* in the pound?

Ans. £429 8*s.* $6\frac{6}{7}$*d.*

41. In what time will a man earn £12 7*s.*, when he receives £$5\frac{1}{4}$ for 8 weeks? *Ans.* $18\frac{86}{105}$ *w.*

42. If a man can complete a certain piece of work in 5 days, what part of it will he do in 3 days?

Part done in 1 day $= \frac{1}{5}$

$\therefore$ „ „ 3 days = 3 times $\frac{1}{5} = \frac{3}{5}$.

43. In what time will a man complete a piece of work, when he can do $\frac{1}{3}$ of it in 2 days?

Time to do $\frac{1}{3}$ = 2 days.

$\therefore$ Time to do the whole = 3 times 2 days = 6 days.

65. The following questions belong to what is called *Double Rule of Three.*

1. If a man travel 200 miles in 10 days of 8 hours long, how many days of 12 hours long will he take to travel 300 miles?

Here, No. hours in travelling 1 mile $= \frac{10 \times 8}{200} = \frac{2}{5}$ hours.

$\therefore$ " " 300 = 300 times $\frac{2}{5}$ hours = 120 hours; and number days 12 hours long $= \frac{120}{12} = 10$ days. *Ans.*

2. If 8 horses consume 7 bushels of oats in 9 days, how many bushels will 17 horses consume in 12 days?

Ans. $19\frac{5}{6}$ bus.

$$\text{No. bus. consumed by 1 horse in 1 day} = \frac{7}{8 \times 9}$$

$$\therefore \quad " \quad " \quad \text{17 horses in 1 day} = \frac{7 \times 17}{8 \times 9}$$

$$" \quad " \quad \text{17 horses in 12 days} = \frac{7 \times 17 \times 12}{8 \times 9}$$

3. If £40 in trade gain £9 in 1 year, what sum will gain £8 in three months? *Ans.* £142 4*s.* $5\frac{1}{3}$*d.*

$$\text{No. pounds to gain £9 in 1 year} = 40$$

$$" \quad " \quad \text{£1 in 1 year} = \tfrac{40}{9}$$

$$" \quad " \quad \text{£1 in 1 month} = \frac{40 \times 12}{9}$$

$$" \quad " \quad \text{£1 in 3 months} = \frac{40 \times 12}{9 \times 3}$$

$$" \quad " \quad \text{£8 in 3 months} = \frac{40 \times 12 \times 8}{9 \times 3}$$

4. If the carriage of 14 cwt. for 9 miles come to £2 9*s.* what must be paid for 2 tons 3 cwt. for 4 miles?

Ans. £3 6*s.* $10\frac{2}{3}$*d.*

5. If 40 acres of grass can be mown by 12 men in 9 days, how many acres will be mown by 16 men in 3 days?

Ans. $17\frac{7}{9}$ acres.

6. If 90*s.* pay 14 men for 7 days' work, how much will it take to pay 22 men for 15 days' work? *Ans.* £15 3*s.* $0\frac{1}{2}$*d.*

7. If £100 gain £2 in 3 months, what sum will gain £4 8*s.* in 11 months? *Ans.* £60.

8. If the weight of a shilling loaf be 80 oz. when the flour is 4*s.* per stone, what will be the weight of a 5 penny loaf, when the flour is 3*s.* per stone? *Ans.* $44\frac{4}{9}$ oz.

9. If 3 men can dig a trench, 30 ft. long and 4 feet deep, in 5 days, how many men will it take to dig a trench, 20 ft. long and 5 deep, in 7 days? *Ans.* $59\frac{11}{21}$ men.

66. *By statement.*

Ex. If 13 yards of cloth cost £4 1*s.*, how many yards can I purchase for £10 9*s.*?

Here, for every £4 1*s.* we have to expend, we shall purchase 13 yards; therefore as often as we can take £4 1*s.* out of £10 9*s.*, so many times 13 yards can we purchase.

$\therefore$ yards required $= \dfrac{£10\ 9s.}{£\ 4\ 1s.} \times 13$ yds. $=$ (bringing the money to the same name) $\dfrac{209 \times 13}{81}$ yds. $= 33$ yds. $2\frac{14}{81}$ qr. *Ans.*

By observing the steps followed in the preceding question we derive the following technical rule.

In this question we require yards for the answer, we therefore put yards for the last term. It is also obvious, from the nature of the question, that the answer must be *greater* than this last term, we therefore put the *greater* of the two remaining terms in the middle, and the less first; we now bring the first and second terms to the same name, then multiplying the last two terms together and dividing by the first, the answer will come out in the same name as the last term. From the property of fractions, proved in Art. **46.** it is obvious, that we may divide the first and second, or the first and third terms, by any number that will divide them both.

$$
\begin{array}{llllll}
\textit{£} & \textit{s.} & \textit{£} & \textit{s.} & & \textit{yds.} \\
4 & 1 : & 10 & 9 & :: & 13 \\
20 & & 20 & & & \\
\hline
81 & & 209 & & & \\
 & & 13 & & & \\
\hline
\multicolumn{6}{l}{81)2717(33\ \textit{yds.}\ 2\tfrac{14}{81}\ \textit{qr.}}
\end{array}
$$

Any of the preceding questions may be solved by this method.

67. *By proportion.*

Ex. If 5 yards cost 9*s.* what will be the cost of 7 yards?

Here the cost will increase with the yards; that is, double the yards, and we double the cost, take the yards 3 times and we increase the cost 3 times, and so on; therefore as many times as 7 is greater than 5, so many times will the cost required be greater than 9*s.*; hence we may put the terms of the question in the form of a proportion.

$$\begin{array}{ccccccc} \textit{yds.} & & \textit{yds.} & & \textit{s.} & & \\ 5 & : & 7 & :: & 9 & : & \text{cost required}; \end{array}$$

by which we mean that the 2nd term is as many times the first, as the 4th term is that of the 3rd. But the 2nd term is 7 times $\frac{1}{5}$ of the 1st term.

$\therefore$ the 4th term $=$ 7 times $\frac{1}{5}$ of the 3rd term.

i. e. cost required $=$ 7 times $\frac{1}{5}$ of $9s. = \frac{7 \times 9}{5}s. = 12s.\ 7\frac{1}{5}d.$

Hence, in a proportion, the 4th, or required term, $=$ the product of the two middle terms, divided by the first.

Any of the preceding questions may be solved by this method.

68. *Questions in Partnership and Division in Ratios.*

1. Two partners, A and B, gain by trade £12; the capital put in by A was £150, and that by B £90; what is their respective shares of the profit?

Here, the total sum in trade is £240, with which £12 are gained; then,

Gain upon £240 $=$ £12
$\therefore$ „ „ £1 $=$ £$\frac{12}{240}$ $=$ £$\frac{1}{20}$
$\therefore$ „ „ £150 $=$ 150 times £$\frac{1}{20}$ $=$ £7 10*s.* A's share.
$\therefore$ „ „ £90 $=$ 90 times £$\frac{1}{20}$ $=$ £4 10*s.* B's share.

2. Two partners, A and B, gain by trade £140; the capital

put in by A was £75, and that by B £94; what is their respective shares of the profit?

Ans. £62 2*s.* $7\frac{41}{169}$*d.* A's, £77 17*s.* $4\frac{128}{169}$*d.* B's.

3. Divide £4 5*s.* 8*d.* between two persons, so that the one may have 3 times as much as the other.

Ans. £1 1*s.* 5*d.*, and £3 4*s.* 3*d.*

4. Divide £14 7*s.* 8*d.* into shares, so that A may have 2 shares, and B 7 shares.

Ans. £3 3*s.* $11\frac{1}{9}$*d.* A's share; £11 3*s.* $8\frac{8}{9}$*d.* B's share.

5. Divide £10 5*s.* between A and B, so that A's share may be $\frac{3}{5}$ of B's.

Ans. £3 16*s.* $10\frac{1}{2}$*d.* A's share; £6 8*s.* $1\frac{1}{2}$*d.* B's share.

6. In 100 ft. of air there are about 20 ft. of oxygen gas, and 80 ft. of nitrogen, how many feet of oxygen are there in an apartment containing 3500 feet? *Ans.* 700 *feet.*

69. *Questions in Simple Interest.*

Interest is the sum paid for the loan of money. The *rate per cent.* is the interest or sum paid for the loan of £100 for 1 year. The sum of money lent is called *the principal.* The amount is the sum of the principal and interest.

1. What is the interest upon £24, at £3 per cent?

Here, interest upon £100 = £3

$\therefore$ „ „ £1 = £$\frac{3}{100}$

$\therefore$ „ „ £24 = 24 times £$\frac{3}{100}$ = 14*s.* $4\frac{4}{5}$*d.*

2. What is the interest of £54 at £2 per cent.?

Ans. £1 1*s.* $7\frac{1}{5}$*d.*

3. What is the interest of £234 at 4 per cent.?

Ans. £9 7*s.* $2\frac{1}{4}$*d.*

4. What is the interest of £423 for 4 years, at 3 per cent. for each year?

Interest upon £100 for 1 y. = £3

$\therefore$ „ „ £1 „ „ = £$\frac{3}{100}$

$\therefore$ „ „ £423 „ „ = £$\frac{3 \times 423}{100}$

$\therefore$ „ „ £423 for 4 y. = £$\frac{4 \times 3 \times 423}{100}$.

From this result we derive the following rule: *multiply the principal, the rate, and the number of years together, and the product divided by* 100 *will be the interest.*

The calculation is usually made after the following form.

$$\begin{array}{rl}
 & \pounds \\
 & 423 \quad \text{principal} \\
 & 3 \quad \text{rate.} \\
\hline
 & 1269 \\
 & 4 \quad \text{no. years.} \\
\hline
1{,}00) & 50{\cdot}76 \\
 & 20 \\
\hline
 & 15{\cdot}20 \\
 & 12 \\
\hline
 & 2{\cdot}40 \\
 & 4 \\
\hline
 & 1{\cdot}60
\end{array}$$

Where we divide by 100 by cutting off two decimal figures from the dividend.

Ans. £50 15*s.* $2\frac{1}{4}d. + \frac{60}{100}f.$

5. What is the interest of £24 for 5 years at 4 per cent. per annum? *Ans.* £4 16*s.*

6. What is the interest of £230 5*s.* for 2 years at 3 per cent. per annum? *Ans.* £13 16*s.* $3\frac{1}{2}d.$ +

7. What is the interest of £160 10*s.* for 3 months at $3\frac{1}{2}$ per cent. per annum? *Ans.* £1 8*s.* $1\frac{1}{20}d.$

In this example the years $= \frac{3}{12} = \frac{1}{4}$.

8. What is the interest of £46 16*s.* $4\frac{1}{4}d.$ for 2 months at $2\frac{1}{2}$ per cent. per annum? *Ans.* 3*s.* $10\frac{3}{4}d.$ +

9. Find the amount of £360 for $4\frac{1}{2}$ years, at 5 per cent.?

Here, the interest = £81; ∴ the amount = £81 + £360 = £441.

10. What will be the amount of £67 for $2\frac{1}{4}$ years, at 4 per cent.? *Ans.* £6 0*s.* $7\frac{1}{5}d.$

11. What is the interest of £35 for 75 days, at 5 per cent.?

Interest upon £100 for 365 days = £5

$$\therefore \quad ,, \quad \pounds1 \quad \text{for} \quad 1 \text{ day} = \pounds\frac{5}{100 \times 365}$$

$$\therefore \quad ,, \quad \pounds35 \quad \text{for} \quad 75 \text{ days} = \pounds\frac{5 \times 35 \times 75}{100 \times 365} = 7s.\ 2\tfrac{1}{4}d. +$$

12. What is the interest of £70 for 21 days at $3\frac{1}{2}$ per cent. per annum? *Ans.* 2*s.* $9\frac{3}{4}$*d.* +

13. What is the interest of £124 3*s.* 2*d.* for 2 years, at $4\frac{1}{2}$ per cent.? *Ans.* £11 3*s.* $5\frac{41}{50}$*d.*

Questions in Compound Interest.

In compound interest, the interest at the end of each year is added to the principal, and then this sum becomes the principal for the next year, and so on.

1. What is the compound interest on £200 for 3 years, at 5 per cent. per annum?

Interest for the 1st year $= \frac{200 \times 5}{100} =$ £10

Then the principal for the next year will be £210

Interest for the 2nd year $= \frac{210 \times 5}{100} =$ £10·5

" 3rd " $= \frac{220{\cdot}5 \times 5}{100} =$ £11 0*s.* 6*d.*

2. What is the compound interest on £75 for 2 years, at 4 per cent. per annum? *Ans.* £3 2*s.* $4\frac{3}{4}$*d.* +

3. What is the compound interest on £60, for 3 years, at 5 per cent. per annum? *Ans.* £3 6*s.* $1\frac{3}{4}$*d.* +

4. What is the compound interest of £50, for 2 years, at 4 per cent. per annum? *Ans.* £2 1*s.* $7\frac{1}{5}$*d.*

70. *Questions in Discount.*

Discount is the allowance which must be made for the *present payment* of a debt, which is not due until a certain time hence.

1. What is the *true* discount upon a bill of £78, due at 1 year hence at 4 per cent.?

Here, the interest of £100 is £4, and therefore the present payment of £104 is £100; that is, the discount upon £104 is £4.

∴ discount upon £1 = £$\frac{4}{104}$

∴ " " £78 = 78 times £$\frac{4}{104}$ = £3.

2. Required the discount, in the last example, when the bill is due at 2 years? *Ans.* £5 15*s.* $6\frac{2}{3}d.$

Here, the interest of £100 for 2 years is £8; therefore, the present payment of £108 is £100, and therefore the discount upon £108 is £8, and so on, as in the preceding solution.

3. Required the true discount upon a bill of £97, due at 1 year hence, at 4 per cent.? *Ans.* £3 14*s.* $7\frac{1}{4}d. +$

4. What is the true discount upon a bill of £450, payable at 2 years hence, money being worth 4 per cent. per annum? *Ans.* £33 6*s.* 8*d.*

5. Required the discount in the last example when the money is to be paid $1\frac{1}{2}$ years hence? *Ans.* £25 9*s.* $5\frac{11}{53}d.$

6. What is the true discount upon £40·5, payable 2 years hence, the value of money being 4 per cent. per annum. Calculate also the discount when the money is payable 1 year hence? *Ans.* £3; and £1 11*s.* $1\frac{3}{4}d. +$

This result shows that the discount is not in the ratio of the time.

7. Calculate the same as in the last example, when the money is worth $3\frac{1}{4}$ per cent. per annum? *Ans.* £2 9*s.* 5*d.*; and £1 5*s.* $5\frac{3}{4}d. +$

71. *Questions in Gain and Loss.*

The loss or gain upon the sale of any goods is usually calculated at a certain rate per cent. upon the *cost price* of the goods.

1. If the gain upon £9 be £2, what is the gain per cent.?

Gain upon £9 = £2

∴ „ „ £1 = £$\frac{2}{9}$

∴ „ „ £100 = 100 times £$\frac{2}{9}$ = £$22\frac{2}{9}$.

2. A draper buys 24 yards of cloth for £6, and retails it at 6*s.* 2*d.* per yard, what does he gain? *Ans.* £1 8*s.*

What is the gain per cent. in this example? *Ans.* $23\frac{1}{3}$.

3. Bought goods for £26, how must they be sold so as to gain 8 per cent.?

$$\begin{array}{lll}
 & \text{Gain upon } £100 & = £8 \\
\therefore & \text{,, ,, } £1 & = £\tfrac{8}{100} \\
\therefore & \text{,, ,, } £26 & = 26 \text{ times } £\tfrac{8}{100} = £2\ 1s.\ 7\tfrac{1}{5}d. \\
\therefore & \text{Selling price} & = £28\ 1s.\ 7\tfrac{1}{5}d.
\end{array}$$

4. A tradesman bought goods for £57, how must he sell them so as to gain 5 per cent.? *Ans.* £59 17*s.*

5. What must be the selling price of goods which cost £32, to yield 25 per cent. gain? *Ans.* £40.

6. If the gain upon £24 be £3, what is that per cent.? *Ans.* $12\frac{1}{2}$.

7. A person bought goods for £64, and sold them for £70, how much did he gain per cent.? *Ans.* $9\frac{3}{8}$.

8. Goods which cost £25 were sold for £30, how much was gained per cent.? *Ans.* 20.

9. A tradesman bought goods for £75, and sold them for £65, what did he lose per cent.? *Ans.* $13\frac{1}{3}$.

10. A person sold goods for £34, and thereby gained 5 per cent. upon the cost price; what was the cost price?

Here, the cost price of £105 will be £100; then

$$\begin{array}{lll}
 & \text{Cost price of } £105 & = £100 \\
\therefore & \text{,, ,, } £1 & = £\tfrac{100}{105} \\
\therefore & \text{,, ,, } £34 & = \dfrac{100 \times 34}{105} = £32\ 7s.\ 7\tfrac{1}{4}d. +
\end{array}$$

11. A tradesman sold goods for £5, and thereby gained 25 per cent., what was the cost price of the goods? *Ans.* £4.

12. What sum or *principal* together, with the interest upon it for 1 year, will produce an *amount* of £46, the rate of interest being at 4 per cent.?

$$\begin{array}{lll}
 & \text{Sum to produce } £104 & = £100 \\
\therefore & \text{,, ,, } £1 & = £\tfrac{100}{104} \\
\therefore & \text{,, ,, } £46 & = \dfrac{100 \times 46}{104} = £44\tfrac{3}{13}.
\end{array}$$

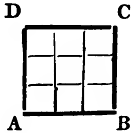
13. What sum or *principal* will produce an *amount* of £35 in 1 year, the interest being at 25 per cent.? *Ans.* £28.

14. The selling price of goods is £48, what was the cost price, the gain being at the rate of 20 per cent. ? *Ans.* £40.

15. What would be the cost price in the last example, if there was a loss of 20 per cent.? *Ans.* £60.

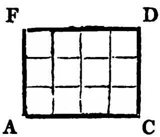
MEASURE OF SURFACES.

72. A surface has length and breadth. When the length is the same as the breadth, the surface is called a square. Thus A B C D is a square, when the length of A B is the same as A D. If the side A B is a yard, the space enclosed by A B C D is a square yard, or a yard of surface; if the side A B is a foot, the surface is called a square foot; and so on.



A square yard contains 9 square feet; because if the sides A B and A D be respectively divided into 3 equal parts, each part will be 1 foot, and lines being drawn through these divisions, as in the figure, we shall have divided the surface into 9 square feet. In the same way it may be shown that a square foot contains 144 sq. inches.

Let the rectangle A C D F be 4 ft. long and 3 ft. broad; then the surface will contain 12 squares, or 12 square feet. For we have 4 squares in each row, and as there are 3 rows, the number of squares in the whole will be 3 times 4 squares, or 12 sq. ft. Hence it appears that the number of square feet in a surface (or area) is found by multiplying the number of feet in the length by the number of feet in the breadth. If the length of the sides be expressed in inches, the product will give square inches; and so on.



Examples.

1. What is the surface in a door 7 ft. long, and 3 ft. broad? *Ans.* 21 *sq. ft.*

2. A table is 5 ft. long, and 4 ft. broad, how many sq. ft. does it contain? *Ans.* 20 *sq. ft.*

3. How many sq. yds. of carpet will it take to cover a floor 8 yds. long, and 5 yds. broad? *Ans.* 40 *sq. yds.*

4. A board is 16 in. long, and 13 in. broad, what is its surface? *Ans.* 208 *sq. in.*, or 1 *sq. ft.* 64 *sq. in.*

5. What will be the cost of paving a court, 24 yds. long, and 16 yds. broad, at 9*s.* for each sq. yard? *Ans.* £172 16*s.*

6. What will be the cost of a door 8 ft. high, and 4 ft. broad, at 1*s.* 8*d.* per sq. ft.? *Ans.* £2 13*s.* 4*d.*

7. A wall is 20 ft. 4 in. long, and 6 ft. 6 in. high, what is its area?

Here, first reducing the dimensions to inches,

20 ft. 4 in. = 244 in., and 6 ft. 6 in. = 78 in.; then

Area = 244 × 78 = 19032 sq. in. = 14 sq. yds. 6 sq. ft. 24 sq. in.

8. What is the area of a floor 18 ft. 2 in. long, and 16 ft. 9 in. broad? *Ans.* 33 *sq. yds.* 7 *sq. ft.* 42 *sq. in.*

9. What will be the cost of building a brick wall 31 yds. 2 ft. long, and 3 yds. 1 ft. high, at 5*s.* 6*d.* per sq. yd.?

Ans. £29 0*s.* $6\frac{2}{3}d.$

10. What will be the cost of flooring a room 18 ft. 3 in. long, and 15 ft. 6 in. broad, at 1*s.* 3*d.* per sq. ft.?

Here, by fractions, 18 ft. 3 in. = $18\frac{1}{4}$ ft. = $\frac{73}{4}$ ft.; and 15 ft. 6 in. = $\frac{31}{2}$ ft. See Art. **52.**

Area = $\frac{73}{4} \times \frac{31}{2} = 282\frac{7}{8}$ sq. ft.

Cost = $282\frac{7}{8}$ × 1*s.* 3*d.* = £17 13*s.* $7\frac{1}{8}d.$ See Art. **61.**

Or thus, by decimals. See Art. **60.** and **59.**

Area = 18·25 × 15·5 = 282·875 sq. ft.

Cost = 282·875 × $1\frac{1}{4}s.$ = 353·59*s.* = £17 13*s.* 7*d.* +

11. What will be the cost of slating a roof 30 ft. long., and 22 ft. 4 in. broad. at 4*s.* per sq. yd.? *Ans.* £14 17*s.* $9\frac{1}{3}d.$

12. What will be the cost of painting a wall 18 ft. 6 in. long, and 10 ft. 9 in. high, at $4\frac{1}{2}d.$ per sq. yd.?

Ans. 8*s.* $3\frac{1}{4}d.$ +

13. What is the area of a square whose side is 9 ft. 6 in.? *Ans.* $90\frac{1}{4}$ *sq. ft.*

14. How many yards length of paper 20 inches broad will it take to cover a wall 16 ft. 4 in. by 10 ft.? *Ans.* $32\frac{2}{3}$ *yds.*

MEASURE OF SOLIDS.

73. A solid that measures a foot every way, that is a foot long, a foot broad, and a foot high, is called a solid foot, or a cubic foot; and so on to a cubic inch, yard, &c.

Examples.

1. How many cubic inches are there in a brick 9 in. long, 4 in. broad, and 3 in. thick?

Here the end of the brick will contain $3 \times 4 = 12$ sq. in.; each inch in the length of the brick will contain 12 cubic inches, and therefore 9 inches in length will contain 9 times 12 c. in., or 108 c. in. It therefore appears that we multiply the three dimensions together to obtain the number of cubic inches or solid content of the brick. Thus, solid content $= 3 \times 4 \times 9 = 108$ c. in.

2. How many inch cubes can be cut out of a block 10 in. long, 5 in. broad, and 4 in. thick? *Ans.* 200.

3. How many cubic feet are there in a block of stone 8 ft. long, 6 ft. broad, and 2 ft. thick? *Ans.* 96 *c. ft.*

4. A piece of timber is 18 in. long, 8 in. broad, and 5 in. thick; how many cubic inches does it contain? *Ans.* 720.

5. How many inch cubes can be cut out of a foot cube? *Ans.* 1728. How many foot cubes can be cut out of a yard cube? *Ans.* 27. How many inch cubes can be cut out of 1 solid yard? *Ans.* 46656.

6. What is the solid content of a block of stone 3 yd. 2 ft. long, 2 yd. 2 ft. broad, and 1 yd. 2 ft. thick?

Here, reducing all the dimensions to feet, we have, 3 yd. 2 ft. = 11 ft., 2 yd. 2 ft. = 8 ft., and 1 yd. 2 ft. = 5 ft.; then solid content $= 11 \times 8 \times 5 = 440$ c. ft. = 16 c. yd. 8 c. ft.

7. How many cubic feet of water will a cistern contain, which is 7 ft. 6 in. long, 4 ft. 6 in. broad, and 4 ft. deep?
Ans. 135 *c. ft.*

8. How many c. ft. of brickwork are there in a wall 30 ft. 2 in. long, 6 ft. 4 in. high, and 1 ft. 6 in. thick?
Ans. 286·583 *c. ft.*

9. What will be the cost of a log of timber 21 ft. 6 in. long, 1 ft. 8 in. broad, and 18 in. thick, at 2*s.* 3*d.* for each c. ft.? *Ans.* £6 0*s.* 11$\frac{1}{4}$*d.*

10. How many c. ft. of timber are there in a joist 17 ft. 3 in. long, 10 in. deep, and 3$\frac{1}{2}$ in. thick? *Ans.* 4·192 c. ft.

11. How many c. ft. of timber are there in 50 spars, each measuring 20 ft. long, 3 in. deep, and 2 in. thick?
Ans. 41$\frac{2}{3}$ *c. ft.*

12. What will be the cost of a block of stone 6 ft. 3 in. long, 2 ft. broad, and 10 in. thick, at 1*s.* 2*d.* per solid foot?
Ans. 12*s.* 1$\frac{3}{4}$*d.*

13. What will be the weight of the stone in the last example, allowing that each c. ft. weighs 140 lbs?
Ans. 1458$\frac{1}{3}$ *lbs.*

14. What will be the weight of a mass of brickwork 10 ft. long, 4 ft. 2 in. high, and 18 in. thick, allowing that each c. ft. weighs 120 lbs.? *Ans.* 7500 *lbs.*

15. How many c. yd. of earth will be cut out of a drain 420 ft. long, 2 ft. broad, and 4 ft. deep; and in what time will a man complete the excavation, allowing that he can lift 500 c. ft. of earth per day? *Ans.* 124$\frac{4}{9}$ *c. yd. and* 6$\frac{18}{25}$*d.*

16. What is the content of a cube whose side is 2 ft. 3 in.? *Ans.* 11·39 *c. ft.*

POWERS AND ROOTS.

74. When a quantity is multiplied by itself it is called the 2nd power or square of the quantity. Thus 6×6, or $6^2 = 36$, is the 2nd power or square of 6. In like man-

ner, three fives multiplied together, or $5 \times 5 \times 5$, or $5^3 = 125$ is called the 3rd power or cube of 5; four twos multiplied together, or $2 \times 2 \times 2 \times 2$, or $2^4 = 16$, is called the 4th power of 2; and so on to other cases.

The square root or 2nd root of a given number is that number which multiplied by itself will produce the given number. Thus the square root of 36, or, as the operation is usually written, $\sqrt[2]{36} = 6$. The cube root or 3rd root of a given number is that number which multiplied by itself 3 times will produce the given number. Thus the cube root of 8, or $\sqrt[3]{8} = 2$; and so on to other cases of roots. It appears, therefore, that the extraction of roots is just the reverse operation of the raising of powers.

Examples.

1. Find the 2nd power of 9; 17; 23. *Ans.* 81; 289; 529.
2. Find the 3rd power of 8; 24; 76.
   Ans. 512; 13824; 438976.
3. What is the 4th power of 13? *Ans.* 28561.
4. What is the square root of 16; 49; 81; $\frac{4}{9}$; $\frac{9}{16}$; $\frac{25}{36}$.?
   Ans. 4; 7; 9; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{6}$.
5. Find the cube root of 27; 64; 512. *Ans.* 3; 4; 8.

75. When the square root of a quantity contains two or more figures the operation must be conducted after the following manner.

1. What is the square root of 205209.

$$\sqrt{2\dot{0}52\dot{0}9\dot{}} = 453$$

```
 √205209 = 453
  16
 ----
 85) 452
     425
     ----
903) 2709
     2709
     ----
     ....
```

Here we first place a point over the units' place, then over every alternate figure. We take the nearest square root of

the 20 (the figures within the first point) which will be 4 with 4 as a remainder. We now bring down 52, the figures included by the next point; and double the 4, the root figure found, which will make 8, this is the *trial* divisor; then 45 divided by 8 gives 5, which is the next figure in the root; we put this 5 also after the 8, and it gives us 85 for the true divisor; we then multiply 85 by 5 and subtract the product from the preceding remainder, leaving 27; we now bring down 09, and multiply 45, the last figures in the root, by 2, making 90 (or what is the same thing we add the 5 to 85) this gives us the new trial divisor; then 270 divided by 90 gives 3 for the next figure in the root; we put this 3 also after the 90, which gives us 903 for the true divisor; and so on.

1. Find the square root of 1681; 4225; 23409; 45369; 18671041; 6; 3.

Ans. 41; 65; 153; 213; 4321; 2·4494+; 1·732+.

76. The novelty of the following rule for extracting the cube root consists in finding the trial divisors by an easy addition, in the place of squaring the root figures, &c., as in the common form. The conciseness, simplicity, and expedition of the method, here proposed by the author, render it highly eligible for elementary instruction.

1. What is the cube root of 93082856768?

$$
\begin{array}{rr}
3\times 4^2 = & 48 \\
3\times 4\times 5 = & 60 \\
5^2 = & 25 \\
\hline
 & 5425 \\
 & 625 \\
\hline
3\times 45^2 = & 6075 \\
3\times 45\times 3 = & 405 \\
3^2 = & 9 \\
\hline
 & 611559 \\
 & 4059 \\
\hline
3\times 453^2 = & 615627 \\
3\times 453\times 2 = & 2718 \\
2^2 = & 4 \\
\hline
 & 61589884
\end{array}
\qquad
\begin{array}{l}
\sqrt[3]{9\dot{3}08\dot{2}85\dot{6}76\dot{8}} = 4532 \\
64 \\
\hline
29082 \\
27125 \\
\hline
\quad 1957856 \\
\quad 1834677 \\
\hline
\qquad 123179768 \\
\qquad 123179768 \\
\hline
\qquad \ldots\ldots\ldots
\end{array}
$$

Here we first place a point over the units' place, and then over every third figure. We take the nearest cube root of 93, which will be 4 with 29 as a remainder. We bring down 082, the figures included by the next point; and take 3 times the square of the root figure, making 48; this is the trial divisor, then 290 divided by 48 gives 5, which is the next figure of the root. To complete the divisor we take 3 times the preceding root figure multiplied by the new figure of the root, and write the product, 60, beneath the 48, taking care to remove the figures a place to the right; in the same manner we write down the square of the new root figure; these added together give 5425 for the true divisor, which multiplied by 5, and subtracted from the subtrahend 29082, gives 1957 as a remainder; and so on to all the other figures of the root. It remains to be shown how the next trial divisor is found without the trouble of squaring 45 and multiplying by 3. Bring down 60 and 25, making 625, and then add the 3 lines of figures connected by the bracket, and this will give 6075 for the trial divisor; and so on to all the succeeding ones.*

* *Proof of the Rule for the Extraction of the Square Root.*

By multiplication,

$$(40+z)^2 = 40^2 + 2 \times 40 \times z + z^2 = 40^2 + (2 \times 40 + z)z;$$

where it appears that, after subtracting 40^2 from the given number, we obtain z, the next figure of the root, by dividing by 2×40, and then for the true divisor we add z, the root thus found, to 2×40. It is evident that this process will hold true for all the successive figures of the root.

Proof of the Rule for the Extraction of the Cube Root.

$$(40+z)^3 = 40^3 + 3 \times 40^2 \times z + 3 \times 40 \times z^2 + z^3$$
$$= 40^3 + (3 \times 40^2 + 3 \times 40 \times z + z^2)\,z;$$

where, after subtracting 40^3, we have for finding z, the next figure of the root, the trial divisor 3×40^2, since the terms which follow this quantity are comparatively small; then to complete the true divisor, to this trial divisor we add $3 \times 40 \times$ the last root figure + the square of the last root figure. The same process must evidently be continued for finding

2. Find the cube root of 12977875; 13824; 76765625; 80677568161; 231; 7; 70.

Ans. 235; 24; 425; 4321; 6·13579; 1·9129; 4·12128.

ON THE CONSTRUCTION OF QUESTIONS,

In which the accuracy of the answers may be tested by an easy operation.

77. The following elegant and really useful method of constructing questions is well deserving the attention of the teacher. It will be observed, that the questions may be formed as quickly as the figures can be written, and that the law of the figures, testing the accuracy of the answer, may be seen at a glance. These features, as far as the author knows, are not to be found in any other method.*

Ex.

```
13,86
34,65
20,79
 5,94
-----
75,24
```

Addition. Here, in each row, the corresponding figures on the left hand added to those on the right produce nines; and the same law is observed in the answer. The only precaution is simply that the sum of the last column should not exceed 9. Nearly all the abstract examples given at page 21. are constructed in this way.

the succeeding figures of the root. It remains for us to show how the trial divisor is found by the easy process given in the rule. For the second trial divisor, for example, we have,

$$3\times 45^2 = 3(40+5)^2 = 3\times 40^2 + 3\times 2\times 40\times 5 + 3\times 5^2$$
$$= 3\times 40^2 + 3\times 40\times 5 + 5^2 + 3\times 40\times 5 + 2\times 5^2.$$

This formula is, in fact, the operation given in the rule. The same process will obviously apply to any other trial divisor.

* The Author gave the demonstration of the rules of construction, with other curious properties of numbers, in the "English Journal of Education," for 1845. This useful periodical is now published by Mr. Bell, Fleet Street.

Ex. 1. 4257
2871
——
1386

Ex. 2. 3245
1265
——
1980

Subtraction. Here the rows are formed in the same way as in addition, and the figures in the answer follow the same law. Instead of the figures, in the question, forming nines, any other number may be selected; in Ex. 2. 7 is the number taken; but here also the figures in the answer follow the law of nines. Nearly all the abstract examples given at page 24. are constructed in this way.

Multiplication. *Ex.* 1. $256{,}9{,}743 \times 34 = 8737{,}1262$.

Here the figures to the left and right of the nine, are formed in the same way as in addition, and the figures in the product follow the same law. When there are 3 figures in the multiplier there must be 2 central nines, and so on. The only precaution is simply that the product of the 2 by the 3 does not produce a number greater than 9.

Ex. 2. $136{,}8{,}752 \times 18 = 2463{,}7536$. Here there is a central 8, the other figures make up eights, the addition of the figures in the multiplier make up 9, and the answer is the law of nines. When the addition of the figures in the multiplier make nine or nines, the figures in the multiplicand may make up any particular number whatever.

Ex. 3. $25{,}66{,}41 \times 264 = 6775{,}3224$. Here the figures in the multiplicand make up some multiple of 3, and the addition of the figures in the multiplier is also a multiple of 3; the figures in the answer have the law of nines. There are *two* central sixes because there are *three* figures in the multiplier, as explained in Ex. 1. Several of the examples given at pages 27., 31., and 32., are constructed in this way.

Division. *Ex.* 1. 51)764,235(14985.

Here the divisor is written at pleasure; the first three figures, in the dividend, are found by multiplying the divisor by 15 and taking 1 from the unit's figure; the remaining figures make up nines, as in addition; then the quotient

figures follow the law of nines, as in the multiplicand of Ex. 1. in multiplication.

Ex. 2. 62; 31)557442(17982. Here to modify the form, we first write down 62 and multiply by 9 (any other number will do), and thus form the dividend as in the last example; then take the half of 62, which gives 31 for the divisor. The figures in the quotient have the same law as in the last example.

Ex. 3. 86; 43)945054(21978.

Ex. 4. 484; 121)33876612(279972.

MISCELLANEOUS QUESTIONS.

1. A person's house rent is £10 8*s.* a year, how much a week must he lay up to pay it? *Ans.* 4*s.*

2. A farmer sold at a fair 15 sheep at £1 16*s.* a piece, and bought 5 yards of cloth at 3*s.* 6*d.* a yard, how much money would he take home? *Ans.* £26 2*s.* 6*d.*

3. A gentleman's yearly income is £464, how much must he spend a week, so as to save £80 a year? *Ans.* £7 7*s.* 8$\frac{1}{4}$*d.*

4. A person pays £18 a year for house rent, £1 16*s.* a month (4 w.) for groceries, and £1 5*s.* a fortnight for bread, how much a week will he save out of a yearly income of £95? *Ans.* 8*s.* 1$\frac{1}{4}$*d.*

5. How many lbs. of tea at 5*s.* 4*d.* a lb. must be given in exchange for 24 lbs. of coffee, at 1*s.* 8*d.* a lb., and 74 cwt. of sugar, at £3 12*s.* a cwt.? *Ans.* 1006$\frac{1}{2}$ *lbs.*

6. A man starts a journey 5 hours before the mail coach, how far will the coach have passed the man, after it has run for 12 hours, supposing that he travels at the rate of 3 miles an hour, and the coach 10 miles an hour? *Ans.* 69 *miles.*

7. The circumference of the fore wheel of a coach is 9 ft., that of the hind one 16 ft., how many more turns will the former make than the latter, in going over a distance of 20 miles? *Ans.* 5133$\frac{1}{3}$.

8. A man walked a journey of 60 miles; for the first 5 hours he walked at the rate of 3 miles an hour, and during the remainder of the journey, he walked at the rate of 4 miles an hour; in what time did he complete the journey?

Ans. $16\frac{1}{4}$ *hours.*

9. A man earns 18*s.* per week, and spends 14*s.* 6*d.* per week; in what time will he save £3 7*s.*? *Ans.* $19\frac{1}{7}$ *w.*

10. A person bought 24 yds. of cloth for £4 3*s.*, at what price must he sell the whole to gain 6*d.* per yard?

Ans. £4 15*s.*

11. A grocer sold a certain number of lbs. of sugar for 6*s.*, at the rate of 8*d.* a lb., and thereby gained 2*d.* a lb.; what was the cost price of the sugar? *Ans.* 4*s.* 6*d.*

12. A person had £7 14*s.* 8*d.* of money. He sold 3 cwt. of sugar at £2 5*s.* 4*d.* per cwt.; and then paid a bill of £5 12*s.*; what money had he left? *Ans.* £8 18*s.* 8*d.*

13. A grocer bought 20 cwts. of sugar for £96; he found that there were 34 lbs. waste. At what price per lb. must he sell the remainder, so as to gain £19? *Ans.* 1*s.* $0\frac{1}{2}$*d.*

14. A person mixed 3 lbs. of tea at 4*s.* 8*d.* a lb., with 5 lbs. at 6*s.* 6*d.*, at what price per lb. must he sell the mixture?

Ans. 5*s.* $9\frac{3}{4}$*d.*

15. If $\frac{5}{7}$ of $\frac{2}{3}$ of an article cost £1 15*s.*, what is the whole worth? *Ans.* £3 13*s.* 6*d.*

16. A corn factor buys 2 qr. 3 bus. of corn at 7*s.* per bus., and 5 qr. at 6*s.*; at what price per bushel must he sell the mixture to gain £3 5*s.* 4*d.* upon the whole?

Ans. 7*s.* $5\frac{2}{59}$*d.*

17. Sound travels at the rate of 1130 ft. per second; what is the distance of a thunder cloud, when the thunder is heard 9 sec. after the flash is seen? *Ans.* 1 *m.* 7 *fur.* 90 *yds.*

18. At what rate per hour must a man travel to complete a journey of 70 miles in 17 hours? *Ans.* $4\frac{2}{17}$ *m.*

19. A fast train starts 3 hours after a slow one. In what time will the former overtake the latter, allowing their speeds to be 40 and 30 miles per hour respectively? *Ans.* 9 *hours.*

20. A cistern has 2 pipes, one of which runs 3 gals. of water per min. into the cistern, and the other 5 gals. per min. out of it; in what time will the cistern be emptied, supposing it to contain 36 gals. at first? *Ans.* 18 *min.*

21. A man's income is 17*s.* 4*d.* per week, and his expenditure is 24*s.*; but he has £15 of money, how long will it support him without getting into debt? *Ans.* 45 *weeks.*

How much will he be in debt at the end of 50 weeks?

Ans. £1 13*s.* 4*d.*

22. How many stones 3 ft. long, and 2 ft. broad, will it take to pave a yard 100 ft. long and 60 ft. broad?

Ans. 1000.

23. A alone can do a certain piece of work in 6 days, and B alone can do it in 4 days, in what time will they do it when working together?

Part done by A in 1 day $= \frac{1}{6}$.

" " B " $= \frac{1}{4}$.

∴ " " A and B $= \frac{1}{6} + \frac{1}{4} = \frac{5}{12}$.

∴ No. days to do the whole $= 1 \div \frac{5}{12} = \frac{12}{12} \div \frac{5}{12} = 2\frac{2}{5}$.

24. A can mow a field in 10 days, B in 8 days, and C in 5 days; in what time will they mow the field when working together? *Ans.* $2\frac{6}{17}$ *days.*

25. If A can do a piece of work in 8 days, and A and B working together can do it in 5 days, in what time would B alone do it? *Ans.* $13\frac{1}{3}$ *days.*

26. A can do a certain piece of work in 7 days, B in 6 days, and C in 5 days. Now A and B worked together for 2 days, when they were joined by C; how many days will they take to complete the remaining portion of the work?

Ans. $\frac{80}{107}$ *days.*

27. A railway company pays 8 per cent. upon the original shares of £60; what rate of interest will I receive, when I purchased shares at £20 premium? *Ans.* 6 *p. c.*

28. A tradesman *lost* 4 per cent., by selling an article for 15*s.*; what should he have sold it for, to *gain* 10 per cent.?

Ans. 17*s.* $2\frac{1}{4}$*d.*

29. A railway share of £30, is to be paid by half yearly instalments of £10; £10 being paid at first, £10 more at the end of the first 6 months, and so on. What will be the worth of the share when it is all paid, allowing that money is worth 5 per cent. per annum, payable half yearly?

Ans. £30 15*s.* $1\frac{1}{2}$*d.*

30. Two men, A and B, rent a field for £14; A puts in 8 horses, and B 50 sheep. What must each pay, allowing that 21 sheep will eat as much as 2 horses?

Ans. A's share = £5 4*s.* $5\frac{1}{2}$*d.*+, and B's = £8 15*s.* $6\frac{1}{4}$*d.*+

Here let us first put the horses into sheep.

$$2 \text{ horses} = 21 \text{ sheep}.$$
$$\therefore 8 \text{ horses} = 4 \times 21 = 84 \text{ sheep}.$$

And altogether we shall have, 84 sheep + 50 sheep = 134 sheep; then

$$\text{Cost for } 134 \text{ sheep} = £14.$$
$$\therefore \quad " \quad 1 \quad " \quad = £\tfrac{14}{134}.$$
$$\therefore \quad " \quad 50 \quad " \quad = £\frac{14 \times 50}{134}, \text{ A's share.}$$

And B's share = £14 − A's share.

31. 3 horses are worth 5 cows, but 4 cows cost £17, how much are 20 horses worth? *Ans.* £141 13*s.* 4*d.*

32. A piece of work can be done by 5 men in 12 hours, or by 4 boys in 20 hours; how long will it take a man and a boy, working together, to do it? *Ans.* $34\frac{2}{7}$ *hours.*

33. If 6 men can reap a field 200 yds. long, and 150 yds. broad, in 4 days of 12 hours each; in how many days of 10 hours long will 8 men reap a field 300 yds. long, and 250 yds. broad? *Ans.* 9 days.

34. A person buys a field of 24 acres at £84 per acre, what should it produce annually to pay 10 per cent.?

Ans. £201 12*s.*

35. A road has a rise of $\frac{1}{6}$ of a foot in 100 ft., what must be the total elevation of the hill, when its length is 2 miles?

Ans. 17·6 ft.

36. The cost price of 50 gals. of wine is £15, but $\frac{1}{5}$ are lost by leakage, and 20 gals. are sold at 4*s.* per gal., at what price per gal. must the remainder be sold, so as to gain 5 per cent. upon the whole? *Ans.* 11*s.* 9*d.*

37. What will be the expense of glazing a window of 16 squares, each 1 ft. 2 in. long, and 10 in. wide, at 4*s.* 6*d.* per sq. ft.? *Ans.* £3 10*s.*

38. A butcher bought 160 lbs. of mutton for £4, and sold $\frac{3}{8}$ of it at the rate of 8*d.* a lb., at what price per lb. must he sell the remainder, so as to gain £2 5*s.* upon the whole? *Ans.* $10\frac{1}{2}$*d.* +

39. Taking a degree upon the earth's surface to be $69\frac{1}{2}$ miles, what distance is London (Lat. $51\frac{1}{2}$°) from the equator? *Ans.* 3579·25 *miles.*

40. A man being asked the hour of the day, said, that the hour after noon was $\frac{1}{5}$ of the time it wanted to midnight. Required the time. *Ans.* 2 *o'clock.*

41. 5 gals. of brandy cost twice as much as 3 gals. of rum; the whole cost £15, what was the price of the brandy per gal.? *Ans.* £2.

42. A pavement is to be laid 260 ft. long, what must be its breadth, so that it may cost £36, allowing 9*d.* per sq. ft.? *Ans.* $3\frac{9}{13}$ *ft.*

44. A can build 7 yds. of wall in 2 days, B 10 yds. in 3 days; in what time will they build 119 yds. working together? *Ans.* $17\frac{17}{41}$ *days.*

45. A draper by selling cloth at 9*s.* 6*d.* per yd. gains 10 per cent.; how much per cent. will he lose by selling the cloth at 8*s.* per yd.? *Ans.* $7\frac{7}{19}$.

46. A bankrupt has liabilities to the amount of £6000; his good debts amount to £2500, and his bad ones to £1600, for which, upon an average, he can only receive 5*s.* in the pound; how much can he pay in the pound? *Ans.* 9*s.* 8*d.*

47. How much sugar at 8*d.* per lb. must I give for 16 lbs. of tea at 4*s.* 6*d.*, so as to gain 10*s.* by the exchange? *Ans.* 93 *lbs.*

48. The length of a railway is 27 miles, and it cost at the rate of £15000 per mile, how many shares of £25 each will the Company have? *Ans.* 16200.

49. How much pure copper can be obtained from 1 ton of the ore, allowing that it yields 6 per cent. of copper?

Ans. 134·4 *lbs.*

50. The rent of a house is £20. It is assessed at $\frac{4}{5}$ of the actual rent. The poor's rate is 8*d.* in the pound, the church rate 6*d.*, and the paving rate 1*s.* 6*d.* What is the total rent? *Ans.* £21 15*s.* 6$\frac{1}{2}$*d.*

51. If the rent of 3 acres for $\frac{1}{4}$ of a year be £4, what will be the rent of 40 acres for 1 year? *Ans.* £213 6*s.* 8*d.*

52. A and B start a journey at the same time and from the same place in opposite directions; A travels at the rate of 4 miles an hour, and B 3$\frac{1}{2}$ miles an hour. In what time will they be 60 miles apart? *Ans.* 8 *hours.*

Supposing the men to travel in the *same* direction, in what time will they be 5$\frac{1}{2}$ miles apart? *Ans.* 11 *hrs.*

53. In a war-ship of 120 guns, there are 50 tons of iron in the form of nails and bolts; what will be the cost of this iron at the rate of 2$\frac{1}{2}$*d.* per lb.? *Ans.* £1166 13*s.* 4*d.*

54. When the mercury in the tube of a barometer is 30 inches high, the pressure of the atmosphere is about 15 lbs. upon every sq. in. of surface; what will be its pressure when the mercury stands at the height of 25 inches?

Ans. 12$\frac{1}{2}$ *lbs.*

55. In the thermometer used in this country the freezing temperature of water is 32°, and the boiling point 212°; in the French thermometer the freezing point is zero or 0, and the boiling point 100°; what degree of the latter will correspond to 60° of the former?

No. Eng. degrees above the Fr. zero $= 60° - 32° = 28°$.

$$180° \text{ Eng.} = 100° \text{ Fr.}$$

$$\therefore \quad 28° \text{ Eng.} = \frac{100 \times 28}{180} = 15\tfrac{5}{9}° \; \textit{Ans.}$$

56. A bought a horse for £20, and sold it to B, thereby gaining 4 per cent.; B then sold the horse for £10 more than it cost him, how much per cent. did he gain?

Ans. $48\frac{1}{13}$.

How much would B gain per cent. if his profit were the same as A's? *Ans.* $3\frac{11}{13}$.

57. A grocer mixes 9 lbs. of tea with 11 lbs. of an inferior quality, and sells the mixture at 6*s.* 6*d.* per lb. Now the first kind was worth 1*s.* a lb. more than the second; required the price of each kind per lb.?

Ans. 7*s.* $0\frac{3}{5}$*d.* and 6*s.* $0\frac{3}{5}$*d.*

58. A sells goods to B for £63, and gains 10 per cent.; then B sells the goods and gains $9\frac{1}{2}$ per cent.; how much more did B gain than A? *Ans.* 5*s.* $1\frac{3}{4}$*d.* +

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